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Prospective teachers' beliefs about problem-solving: Cypriot and English cultural constructions

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In this paper we report on a small-scale comparative examination of prospective elementary teachers' beliefs about problem-solving in Cyprus and England. First year undergraduate students (13 from Cyprus and 14 from England) from a well-regarded university in each country were qualitatively interviewed at the commencement of their respective teacher education programmes. Data, which were analysed by means of a combination of theory- and data-driven coding, indicated that, in both countries, students entered university with beliefs about problems and problem-solving that were not only products of the cultures in which they were educated, but also frequently incommensurate with the problem-solving expectation of the curricular frameworks within which they would have to work as teachers. Also, the outcomes confirmed that, despite researchers' assumptions of definitional convergence, the expressions 'mathematical problem' and 'problem-solving' continue to be used differently across cultures. Some implications for teacher education are discussed.

Keywords: Problem-solving; beliefs; teacher education

Introduction

Problem-solving, as a learning outcome, is emphasised in the curricula of most countries. Yet, how outcomes are realised depends on teachers' abilities to provide appropriate opportunities for students to acquire such skills and their dispositions to do so. In this paper, we examine the beliefs about problem-solving of beginning undergraduate primary teacher education students in Cyprus and England. Entering university directly from school, these students are products of the opportunities to learn afforded them by their teachers. As beginning undergraduates, they are to enter a new stage as conceptualised by those working in teacher education. Before and after entering university, students are located in a cultural context with regard to didactic traditions and systemic expectations. This paper attempts to examine the beliefs about problem-solving that prospective teachers bring from school to teacher education. Additionally, a comparative focus allows us to demonstrate the extent to which these students' beliefs are culturally informed. However, neither the word *problem* nor its allied term *problem-solving* is clear enough in the literature.

The nature of mathematical problems and problem-solving

In the mathematics education literature, the use of *problem* typically assumes that readers have shared understanding of the term (Lester 1994; Schoenfeld 1992).

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Historically the word distinguishes those who understand problems as routine exercises for the practice and consolidation of newly taught procedures from those who view them as tasks whose difficulty makes them genuinely problematic. In the latter case, a mathematical problem presents an objective with no immediate or obvious solution process (Polya 1945; Blum and Niss 1991; Nunokawa 2005). Consequently, a key characteristic of a mathematical problem is the person solving it; the complexity of a problem is a function of the knowledge, experience and dispositions of the solver (Borasi 1986; Blum and Niss 1991). As Schoenfeld (1985, 74) notes, "being a 'problem' is not a property inherent in a mathematical task. It is a particular relationship between the individual and the task that makes it a problem for that person". This relationship, we suggest, could be summarised as follows: individuals must accept an engagement with the problem they intend to solve; they must have encountered a block and see no immediate solution process; and they must actively explore a variety of plausible approaches to the problem. Lastly, we acknowledge that mathematical problems may be located in a real, a mathematical (Blum and Niss 1991) or a realistic world (Van den Heuvel-Panhuizen 2003).

The majority of the world's mathematics curricula and recent international studies, such as the Programme of International Student Assessment (PISA) (OECD 2001, 2004, 2007) highlight the importance of problem-solving as a key element of pupils' learning. However, if the literature is unclear in meaning then one could assume similar ambiguity with regard to problem-solving (Arcavi and Friedlander 2007). Such ambiguity does not imply that problem-solving is not construed as an integral element of students' mathematical learning; it simply reflects differences in how the subject is conceptualised. Indeed, there are various uses of the notion in the literature (Chapman 1997), although prevailing perspectives appear as process, curricular goal, and instructional approach. Therefore, we focus our review on these. In so doing we acknowledge that we have overlooked other aspects of problem-solving; for example, the cognitive processes employed by solvers of word problems that are well rehearsed in the mathematics education literature.

Problem-solving as a curricular goal

For many educational systems, problem-solving is a primary goal of the intended curriculum. For instance, it is one of the five fundamental mathematical processes identified by the National Council of Teachers of Mathematics (NCTM 2000) and, in consequence, a key objective of many US states' curricula. In many European nations, problem-solving and its related skills form key expectations of the intended curriculum for students of all ages. Similarly, the Singaporean authorities have placed problem-solving at the centre of mathematics as a school subject, as seen in the familiar pentagonal model presented in that country's intended curriculum (Fan and Zhu 2007). In short, many educational systems, through the medium of the intended curriculum, have highlighted problem-solving and associated skills, as important outcomes of the learning process. However, the means by which such emphases are expressed vary. For example, Cai and Nie (2007) suggest that the Chinese and US curricula expectations differ substantially. The Chinese present problem-solving as a key learning outcome, while the US, as reflected in the NCTM's standards, emphasise problem-solving as outcome and instructional approach.

Problem-solving as a process

According to Cai and Lester (2007, 221) problem-solving "is an activity requiring the individual to engage in a variety of cognitive actions, each of which requires some knowledge and skill, and some of which are not routine". Over the years, various writers have developed frameworks for analysing and describing the components of such activity. For example, Nunokawa (1995) writes that mathematical problem-solving comprises three phases; formulating the problem into mathematical terms, operating within the formulated mathematical world, and translating the mathematical results into the original context. Others have offered frameworks that describe and structure the process of problem-solving. The best known is Pólya's (1945) four-phase model; understand the problem, devise a plan, implement the plan, and reflect. Indeed, most subsequent models have refined Pólya's, as in Mason, Burton, and Stacey's (1982) three phase contracted model, or Kapa's (2001) six-phase expanded one. However, Pólya-style models have been prone to misinterpretation as linear applications of a series of steps (Kelly 2006; Nunokawa 1994). Of course, the very nature of mathematical problems requires solvers to be able to transform problems before applying existing knowledge (Nunokawa 2005). In such acts of transformation, solvers may acquire new insights and methods may emerge to augment their mathematical repertoires. This process of acquisition alludes to a second major goal of problem-solving-related mathematics instruction.

Problem-solving as an instructional approach

Schroeder and Lester (1989) distinguish between teaching *for* problem-solving and teaching *about* problem-solving. Through the exploitation of existing knowledge, teaching about problem-solving deepens students' understanding of the problem context and facilitates students' learning of "mathematical content and how that new content is related to the mathematical knowledge they already have" (Nunokawa 2005, 332). Moreover, mathematical problem-solving provides opportunities for students to enhance their flexible and independent mathematical thinking and reasoning abilities (Cai and Nie 2007). However, irrespective of the intended outcomes, the transfer of students' mathematical knowledge and skills to problem-solving situations is problematic, often due to students' inadequate understanding of the structure of mathematical problems (Bassok 1990), or to the manner in which teachers collude in the construction of inflexible knowledge (Bransford, Brown, and Cocking 2000). Therefore, we are aware that the achievement of problem-solving-related goals is complicated, and that much teaching, frequently focused on the acquisition of conceptual and procedural knowledge, is unlikely to have any significant impact on them (Silver et al. 2005).

Problem-solving and teachers' beliefs: a culturally located relationship

The relationships between the affective domain (i.e. emotions, attitudes, beliefs) and mathematical problem-solving are well researched. Yet, most studies in this area are concerned with the affective dimensions of learners' problem-solving processes, as well as the interrelation between knowledge, control, affect, beliefs and context (see for example Goldin 2004; Cai 2010; Op't Eynde, De Corte, and L. Verschaffel

2003, 2006). While acknowledging these as important for locating our work in the wider problem-solving research territory, further discussion on affect in general would go beyond the scope of this paper, which focuses on prospective teachers' beliefs.

There is a growing acceptance that what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom (Ernest 1989; Thompson 1984; Wilson and Cooney 2003). Thus, understanding the relationship between beliefs and practice should deepen our understanding of the teaching process (Aguirre and Speer 1999) and facilitate better-focused teacher education programmes (Cooney, Shealy, and Arvold 1998). Despite such ambitions, this relationship has been shown to be complex and rarely amenable to cause-and-effect accounts. For example, while several studies have highlighted substantial disparities between espoused and enacted beliefs (Thompson 1984; Beswick 2005), others have indicated that both beliefs and actions are contingent on the changing nature of the classroom context (Schoenfeld 2000; Skott 2001).

Despite uncertainty with respect to the espoused-enacted relationship, there is evidence that serving teachers' mathematics-related beliefs (Santagata 2004; Correa et al. 2008) and practices (Givvin et al. 2005; Andrews 2007a) are substantially determined by the cultural context in which they operate. That is, culture plays a key determinant role in both belief formation and manifestation. In this respect, Andrews (2011) has written that teachers work within three curricula; an intended, an idealised and a received curriculum. That is, they are constrained (or liberated) by the systemic curricular model within which they operate; they approach their work with a unique set of articulable personal goals, and they work within cultural norms that may be beneath articulation.

With regard to problem-solving, the cultural dimensions of teachers' beliefs and practices appear to be under-researched. Nonetheless, there is rising evidence that the cultural context in which students are raised and taught mathematics considerably effects their ability to solve non-routine mathematical problems. For example, Cai (1995) highlighted significant differences between Chinese and US pupils' performances in problem-solving tasks. These differences were later shown to be a consequence of choices teachers made with regard to the problem-solving representations that they privileged (Cai and Lester 2005, 2007). From a European perspective, a recent study of mathematics teaching in four European countries found four times as many problem-solving episodes in Hungary than in Flanders (Andrews 2009). Such studies confirm that classroom norms in general, and problem-solving in particular, are culturally determined. In the light of these, this paper is an attempt to further our understanding of the ways in which a teacher's (or prospective teacher's) beliefs about problem-solving are influenced by the cultural context in which they operate, through examination of the beliefs of prospective teachers in Cyprus and England.

Primary mathematics curriculum goals in these two cultures

By way of framing our study, it seems appropriate to summarise the respective systemic expectations with respect to problem-solving. Both the Cypriot and the English educational systems are currently developing reforms for all school subjects. According to the Ministry of Education and Culture of Cyprus (MOEC) and the

Department for Education of England (DfE), the new curricula will be implemented after September 2011 in both countries (see the websites of MOEC and DfE). Therefore, we focus our discussion here on the 1999 primary mathematics curricula of Cyprus and England currently being implemented. In its introduction, the Cyprus national curriculum for primary mathematics (Curricula Development Service1999) asserts its curricular aims as, inter alia, the "development of students' investigative spirit, conceptual understanding of procedures, discovering skills though personal experiences and problem-solving activities" (101). With regard to mathematics, it continues by suggesting that the purpose of "teaching mathematics is to enable students to identify and solve mathematical problems which are useful in daily life and in other sciences" (101). In addition, for each grade of primary education, the various topics to be taught are presented alongside. In this list of topics, references to Pólya's problem-solving phases and strategies are included. Finally, the document asserts that "problem-solving is not a separate unit of mathematics. It is included in the curriculum as a separate sub-topic, in order to highlight its significance", and permeates "every unit of mathematics" (115).

The generic statements of the English curriculum assert that:

> mathematics provides opportunities for pupils to develop the key skills of ... *problem-solving,* through selecting and using methods, developing strategic thinking and reflecting on whether the approach taken to a problem was appropriate (DfE and QCA 1998, 9).

Shifting from an emphasis on mathematics itself, the document adds (9) that "mathematics provides opportunities to promote ... *thinking skills,* through developing pupils' problem-solving skills and deductive reasoning" and "*work-related learning,* through developing pupils' abilities to use and apply mathematics in workplace situations and in solving real-life problems". With regard to the content knowledge incorporated in the document, mathematics is presented within two or three broad content areas, which include expectations that students engage in problem-solving and various related strategies related to Pólya's three phases, and concerns with the identification of appropriate mathematics, strategy selection and implementation. As far as the systemic rhetoric is concerned, problem-solving is an integral element of the Cypriot and English learning experience, with similar emphases. However, this paper examines whether such systemic ambitions are met in undergraduates' experientially- and culturally- derived beliefs about problem-solving.

Methods

General research approach

Research on mathematics teachers' beliefs has tended to fall into two camps. On the one hand are survey studies, frequently involving factor analytic techniques to identify or confirm belief-related constructs (Beswick 2005; Staub and Stern 2002). Such approaches, while clearly important, rely on predetermined categorisations and, of course, are not designed to attend to the voice of the individual. They may miss not only subtle variations within the beliefs components under scrutiny, but also components hitherto unconsidered. Moreover, such approaches tend to lack explanatory power, being focused more on *what is* rather than *why*. On the other hand are case-studies designed to uncover the complexity of individual prospective and in-service teachers' beliefs and practices (Thompson 1984; Cooney, Shealy, and Arvold 1998; Aguirre and Speer 1999). Such studies highlight the relationship between beliefs, as espoused by informants, and practice, as observed. However, while rich in detail, they lack generalisability, unless viewed as elements of a critical mass of similar studies.

Our approach falls between the two camps. We have sought to address, albeit in a small exploratory study, issues of generality by working with, in relation to the cohort size, representative samples of students. We have also attempted to account for the individual's voice by inviting our informants to tell their own stories about mathematical problems and problem-solving by means of the traditions of narrative research. Narrative research, which privileges the individual's voice, seeks to understand the ways in which informants construct stories to make sense of their professional world (Swidler 2000; Drake 2006).

Participants and data collection

Two volunteer groups of Cypriot and English first year undergraduate elementary education students were interviewed by the first author. Their universities are well-regarded, according to various national measures, for their teacher education programmes. Our objective was to elicit their perspectives on the nature of mathematical problem-solving at the point of entry to their respective courses. As such, having not been exposed to university perspectives, these could be construed as indicators of the extent to which systemic curricular expectations had been met. Also, their perspectives could be seen as baseline assessments for their courses. The Cypriot cohort comprised thirteen students (twelve female, one male), while the English comprised fourteen (thirteen female, one male). The gender ratio of both samples reflected the gender ratio among undergraduates in both departments. The interview questions were based on an amended version of Ernest's (1989) model on teachers' beliefs. Specifically, in addition to the dimensions about the nature of mathematics, its teaching, and its learning, two extra dimensions, concerning self-efficacy about mathematics and its teaching, were introduced, giving us five dimensions. Participants in both countries were interviewed in the first week of their courses, before any official mathematics-related university instruction. The semi-structured interviews (audio-recorded and later transcribed) did not exceed 35 minutes, and took place at both universities. Examples of questions from the interview protocol are presented below:

(1) What is a mathematical problem to you?
(2) What characteristics should a good problem have?
(3) Can you give examples of mathematical problems from your personal experiences?
(4) What does mathematical problem-solving mean to you? Can you define it?
(5) What are the similarities and/or differences between an expert and a novice solver?
(6) What should someone do, in your opinion, in order to improve her/his problem-solving skills?

Data analysis

Conventionally, analyses of qualitative data are either theory-driven or data-driven (Boyatzis 1998; Kvale and Brinkmann 2009). The former exploits codes developed prior to the analysis, usually from the extant literature, while the latter generally derives codes iteratively by the constant comparison process outlined by Strauss and Corbin (1998). In this project neither was employed, because the former would preclude students' voices from emerging, while the latter would prevent comparisons between the two cohorts, due to the emergence of themes and categories unique to each country's students. Consequently, a compromise was made, involving a combination of theory- and data-driven coding, in a phenomenographic manner (Marton 1981; Cope 2004). In other words, we were interested in the ways participants in each country talked about the five dimensions of the amended model on teachers' beliefs, presented above. Initial readings of the interviews have led us to the generation of an eight-theme framework. The broad and neutral themes around which further analyses were based are: (1) nature of mathematical problems (2) nature of mathematical problem-solving (MPS) (3) the expert problem solver (4) affective issues of learning (5) cognitive/metacognitive issues of learning (6) the self as solver (7) explicit pedagogic practice, and (8) the self as teacher. Sub-themes relating to each of the eight themes were derived from the data. The two data sets, Cypriot and English, were analysed separately in accordance with these themes, to ensure that culturally-located differences were not obscured. In this paper, the first three themes are presented and discussed. These themes can be construed as, essentially, definitions. Unlike the remaining five themes, they include no judgments about the individual and her or his competence as either solver of problems or prospective teacher. In this respect, they represent a self-contained set of perspectives. Importantly, as our participants were recent school leavers, they had experienced neither university instruction nor classroom practice as teachers. Therefore, on the one hand, their beliefs about the nature of mathematical problems and problem-solving, as well as their perceptions of the characteristics of expert solvers, were well-formulated products of their prior schooling. On the other hand, as beginning teacher education students, they were not expected to have well-formulated self-efficacy beliefs, particularly with respect to their own teaching competence.

Results

In the following pages, the Cypriot results are presented.

CY: The nature of mathematical problems

Problems constitute clear verbal descriptions of mathematical situations

The comments of all thirteen first year Cypriot students indicated that mathematical problems were defined more by their form than their function. For example, Panayiota commented that mathematical problems comprise a set of "mathematics-related sentences, which include information, data and a desired outcome. We have to think about the data, to process them and get the answer". She added that problems should be neither too long nor use vague language so that "everyone would be able to understand them after reading them a couple of times". This sense of clarity was mentioned by all students in this group. Sofia, for example, commented that a good mathematical problem "should be clearly presented, with clear instructions. It should give you enough data in order to be able to solve it".

Problems are inherently difficult

Problem difficulty was mentioned by eight students. Anastasia observed that "problems can be categorised as easy or hard", with the former requiring only one step for reaching an answer and the latter being more complicated. This perspective was reflected by Christina's comment that 'easy problems are for weak students, while hard problems have more numbers and symbols'. Others added that difficulty was an essential characteristic of problems, as seen in Demetra's comment, "if it is easy, surely I won't consider it a problem". Haroula argued that problem difficulty was connected to student age, commenting that, "in primary school problems are very easy, in gymnasium they are more complicated, and in lyceum you find the hardest". All the above indicate that problems are intended to be difficult, as reflected in Sofia's comment that a problem, by definition "has difficulty and unknown factors".

The context of problems: real-world situations

All thirteen students alluded to a belief that mathematical problems are contextualised within real-world frameworks. Specifically, all of them referred to the so-called consumer problems, which, according to Panayiota, are about "capital, interest, taxes, V.A.T., percentages, discounts and so on". Martha provided an example of a consumer problem: The initial price of a t-shirt is that, and the discount is, let's say 20%. We have to find the new price of the t-shirt, after the discount. Christina, in turn, provided some examples from primary and upper-secondary (lyceum). She said:

> I remember in primary school we were given problems like "a farmer has so many orange trees, so many apple trees" . . . In lyceum, because I had taken core mathematics and not enhanced, I didn't encounter many problems. I had simple consumer problems, V.A.T, percentages, the kind of stuff related to everyday life.

CY: The nature of mathematical problem-solving

MPS is a process linking the given and the desired

Eleven of the thirteen students saw MPS as a process whereby the solver interacts with the problem in order to solve it. Stefania, for example, said that MPS is "the way towards the solution, the stages you follow for resolving the problem". Pantelis, in comments typical of those concerning the separation of what is known from what is desired, said that "mathematical problem-solving is a process, the process towards what we are asked to find. It is the process during which you use the given data in order to find the answer to a problem". All eleven students commented that successful problem-solving requires repeated reading of the problem, the extraction of data, choosing relevant information, and so on. Panayiota's comment was typical of others. She said:

> You have to read the problem two or three times, underline some key points, because you know, sometimes problems have unnecessary things in them, you have to find what is important, then start processing all this in your mind, read it two to three times, write down your data and what you want to find, do a shape if it's needed and then do the algorithms.

CY: Expert problem solvers

Intellectually nimble, though practice helps

All thirteen students indicated that good problem solvers are perceptively quick, able to recognise what is needed, concentrate hard and are organised. Sofia commented, on the one hand, that "as soon as a good solver sees the problem, he has a clear picture in mind about what has to be done immediately". On the other hand, someone "who is less adept … will have difficulties in finding which way to follow for solving the problem". A key characteristic of an expert is concentration, according to Demetra. She said that an expert is:

> one whose mind works really quickly … in order to solve a problem you should be able to understand it the first time you read it. So, someone who is not concentrating one hundred per cent at the time of reading it will face more difficulties. Therefore, a good problem solver is the one who concentrates and understands the problem quickly.

For a small number, such skills were innate rather than acquired. For example, Panayiota commented that problem-solving competence "depends on the individual's innate talent … We say 'he is born to do mathematics' or 'his brain is square'[1], something like that. I think that's it. Some people are born with it … Let's say, you either have a knack for it or not".

All students indicated that such skills could be acquired through practice. Demetra's perspective was typical: "I wasn't a good problem solver, but now I consider myself a very good one. Practise, I believe. Train your mind and then you will familiarise yourself with problems". On a slightly less optimistic note, Haroula commented that:

> a weak problem solver could improve a little bit, if he had guidance from a good solver; and although he won't become expert, at least he would be able to solve some problems to a satisfactory extent.

In the following pages, the English results are presented.

EN: The nature of mathematical problems

Number-related mathematical situations

Ten of fourteen students described mathematical problems as situations that required number operations. For example, Victoria commented, "it is anything, from adding, dividing, subtracting, multiplying, or arranging and putting them together", while Laura indicated that solving problems is the same as "solving numbers, differences or similarities between numbers". Considering others' comments, Rachel alluded to a broader conception. She said that a mathematical problem could be "anything to do with numbers really . . . Something to do with quantity or number, a problem could be, how many bags of sugar or, em, eight times something equals something else".

The context of problems: real world, mathematical world or both

Unlike their Cypriot colleagues, the English cohort appeared more fragmented in its perspectives on the context of mathematical problems. A group of four students saw problems as located in a world of mathematics. For example, Daniel, recalling his own experiences of pre-university mathematics, said:

> It's been a while . . . can't remember how it was, like we've got sine, cosine and tangent and you need to work out the other two. You've got one of the angles and you need to work out another angle or length. That's one which sticks in my head, it is quite difficult.

Two students argued that mathematical problems are situations related to real world and everyday life. For example, Melanie described situations:

> Regarding the bus timetable, something like that. So, if you need to be somewhere, which bus do you have to catch? How long will your journey take? Do you have to change buses and why? Take the time into consideration.

However, the majority view expressed by the remaining eight students was that mathematical problems could be embedded in either world. As Victoria commented, "you can have [problems] related to real life situations and you have ones that are just mathematics". Similarly, Margaret suggested that the same problem could be presented in either context. She said that:

> We were taught about Pythagoras' theorem and, em, we were taught a lot of problems, like 'you need a ladder to put against the wall' and it had to be a certain distance between the wall, and the ladder was a certain length, so how far from the wall would you go. That kind of, em, applied theorems to real life situations to make them more real . . . The same problem could also be presented as an abstract one. Em, if one side is three centimetres, one is four centimetres, what's the longest?

EN: The nature of mathematical problem-solving

MPS is a structured process exploiting prior knowledge

Eleven students described MPS as a structured, step-by-step process. Julia's response was typical. She said that MPS was "just basing what skills you know on trying to solve a problem in maths, so just applying the knowledge to structure it and work step by step to work out a problem". This step-by-step approach was associated with breaking down a problem into smaller tasks, and working on each separately before putting them back together. For example, Laura commented that problem-solving required the solver to "take it step by step and apply things you already know". Similar comments were made by Victoria. She said:

> If it was me, I would have to read the question and then I'd have to take the question apart. And then, once I've got the numbers sorted in my head, I am not very good at

> maths so, I have to sit there for ages. And then, once I've got them sorted I try to think of what I need to do first and what I need to move on.

EN: Expert problem solvers

The expert solver knows the formulas and strategies

Thirteen students indicated that an expert problem solver knows the mathematical formulas and strategies necessary for approaching a problem. Laura, for example, suggested that:

> An expert knows how to answer straight away, whereas someone who is not so proficient thinks a long time about it and may only have one option, whereas an expert might have lots of different ways to think about it".

Expert solvers, she added, "already have the knowledge to work out what they need to do to solve problems, otherwise they have to think of the steps to take". Similarly, Georgina commented that:

> Experts have … all the techniques they can apply to each problem or adapt to a different problem, but someone who doesn't know what they're doing just looks at numbers that don't mean anything.

Practice makes perfect

The same thirteen students also suggested that problem-solving skills could be improved with practice. As Daniel observed:

> I would say practice. Practice makes perfect. Em, the more you practise, the more confident you'll be with dealing most of the problems. Em and the more you practise, it is proven psychologically, it will be embedded in your brain.

Daniel's view was reflected in the comments of others. Rachel suggested "I think practice makes perfect', while Jenny said:

> 'I'd say with maths it is practice. It is not an either/or situation; it is not you are good at it or you aren't; you've got to try and practise, that's it! And the more you practise the more you get. I mean, practice is basic in maths".

Discussion

The data presented above allude, across the board, to both similarities and differences in the problem-solving beliefs espoused by these two cohorts of first year undergraduate teacher education students. Below, each of the three broad themes presented above is discussed separately before some final thoughts are offered.

The nature of mathematical problems

For the Cypriot students, a mathematical problem was something clearly presented in a verbal context from which the solver needs to extract and analyse information in order to resolve it. Such a perspective resonated closely with others' descriptions of mathematical word problems (Jonassen 2003). English students, who did not mention words in their descriptions, defined a mathematical problem as a task that involves numbers. Perhaps unsurprisingly, this coincides with the results of an earlier study which found English secondary mathematics teachers arguing that the importance of mathematics lay in the teaching of all aspects of number and its applications (Andrews 2007b).

Unlike their English colleagues, the Cypriot students expressed a particular belief that not only should problems be transparent, but they should also be presented in ways whereby the problem statement ends with a clear articulation of the problem solver's goal. This view conflicts with the more generally accepted perspective that problem goals, or formulations, may either be explicit in the text, or something for the solver to derive from it (Borasi 1986). It is possible that such views reflect hidden cultural norms. For example, Hofstede (1986) has suggested, inter alia, that cultures differ in the ways they deal with uncertainty. His analyses indicated that the English are relaxed with ill-defined situations, whereas Greeks, a close cultural neighbour to Greek Cypriots, are among the most uncomfortable of all nationalities with uncertainty. In other words, Greek people appear uncomfortable with undefined situations. Such characteristic behaviours may not only explain why Cypriot students expect problems to be well-defined, leaving no space for ambiguity, but also why English students do not. In a similar vein, the Cypriot students tended to argue that problems are inherently difficult for the learner, a perspective largely absent from the English.

All the Cypriot students located their interpretation of mathematical problems in some sense of the real world. Such a perspective may be due to a systematic inclusion of real-world verbal tasks in the National Textbooks of Cyprus. Indeed, several Cypriot students spoke of consumer problems, which are the only tasks explicitly labelled as problems in the lyceum textbook. In contrast with their Cypriot peers, and despite their restrictive belief that problems are always number-based, the English participants appeared to hold more eclectic beliefs than the Cypriots about the context of problems. Specifically, the majority indicated that problems could be either purely mathematical or real-world related, a perspective commonly agreed internationally (Blum and Niss 1991; Borasi 1986). This perspective is not unlikely when one considers the sorts of problems found in English texts and the various emphases, both real world and mathematical, therein. In short, students' perspectives on the context of problems, and the topics embedded in them, appear to be culturally-informed constructions.

The nature of MPS

Both cohorts described MPS as a process, although each characterised the process differently. The Cypriot students discussed a holistic – read, understand, collect data, analyse data and so on – sequence of actions in the manner reported by Nunokawa (2005). Moreover, the Cypriot students indicated that the problem-solving process involves multiple readings of the problem text, a view consistent with research highlighting the importance of understanding the language structures of word problems before attempting a resolution (Cummins 1988; Adetula 1990), as part of understanding (Polya 1945) or gaining entry into the problem (Mason, Burton, and Stacey 1982). Interestingly, the majority's separation of the given information from the desired outcome may have its root in traditional Cypriot teaching, whereby learners are encouraged, at both primary and secondary levels, to make a two-column table with the given information necessary for the solution on one side and the desired outcome at the other. This very plausible explanation provides further evidence that prospective teachers enter teacher preparatory programmes with predetermined ideas about mathematics and its teaching derived from their prior schooling experiences (Kagan 1992; Cooney, Shealy, and Arvold 1998).

The English students saw the MPS process as one of simplification or reduction of the task to a series of small steps. Such a perception is not inconsistent with misinterpretations of Polya's problem-solving stages as a linear series of steps (Nunokawa 1994; Kelly 2006). Also, their emphasis on simplification resonates with earlier descriptions of English teaching as a reduction from the complexity of mathematics (Jennings and Dunne 1996), or as being pragmatic (Kaiser, Hino, and Knipping 2006). Moreover, the English cohort seemed to construe problem-solving as something undertaken after new concepts and procedures have been taught. That is, teaching *for* problem-solving (Schroeder and Lester 1989). Interestingly, with one exception in each cohort, all students used circumlocution to describe problem-solving as a process. Thus, it could be argued, that the students of both cohorts entered their teacher education programmes with a naïve vocabulary regarding the concepts of their profession.

The expert problem solver

Just over half the Cypriot cohort described an expert problem solver as someone who is able to interpret a problem quickly, and then plan and implement a solution strategy effectively. For all English students, the expert solver knows mathematical formulas and strategies and can apply them to a problem in accordance with their perspectives on problem-solving as the application of prior learnt knowledge. Both cohorts' perspectives find resonance in Guilford's (1956) theory of the structure of intellect. The Cypriot view accords with an emphasis on *cognition* or the ability to understand, comprehend, discover, and become aware of information. Such views seem characteristic of Muir, Beswick and Williamson's (2008) sophisticated problem-solving behaviour. The English cohort privileges memory retention or the ability to recall information, in accordance with Muir, Beswick and Williamson s (2008) sense of routine behaviour. Interestingly, no student, irrespective of cohort, commented in ways that would indicate an emphasis on any of the remaining four of Guilford's intellectual processes; memory recording, divergent production, convergent production or evaluation. In respect of the English cohort, this singular belief may be linked to the values underpinning England's educational system; Anglo-Saxon educational traditions typically promote rule memorisation at the expense of the development of critical thinking (Haggis 2003; Entwistle and Peterson 2004). The only significant difference between the two cohorts was found in the few Cypriot students who, despite arguing that good problem solvers are genetically determined, conceded that practice could still improve an individual's performance.

Conclusions

Each cohort of students, while clearly limited with respect to representation, presented consistent beliefs with respect to the three themes discussed above. In summary, it seems unlikely that such collective beliefs could have arisen in isolation of their experiences as learners of mathematics. Of course, the students interviewed for this study were not teachers. They were at the beginning of their undergraduate courses, untainted by the expectations of their respective teacher education traditions. Therefore, their beliefs were likely to be espousals of both their articulable idealised curriculum and, more importantly, their inarticulable received curricula (Andrews 2011). Students had assimilated a particular set of experientially-informed perspectives on, and approaches to, problem-solving, which they accepted as the natural state of things, having no other points of reference.

Yet, because they were at the start of their teacher preparation courses, these students had never seriously anticipated engaging with the curriculum in the manner expected of a teacher. They had only experienced the curriculum through the mediating beliefs of their own teachers, and to a lesser extent, their parents and wider society. Therefore, if students' beliefs were at variance with curriculum expectations, then the same is likely to have been true of their teachers. Such variance indicates that the teachers of our students must have been working within either idealised or received curricula, frequently incommensurate with the intended. However, the extent to which students' beliefs resonated with their national peers suggests that their teachers were operating within similar received, rather than idealised, curricula. Thus, the received curricula of teachers impact considerably on students' espoused beliefs about problems and problem-solving.

In conclusion, if prospective teachers' beliefs about problem-solving are manifestations of their respective received curricula, and the data above support such a conjecture, then it behoves those responsible for teacher education programmes to devise interventions to address such cohort-wide divergences from the intended curricula. Not to do so would be to collude in the maintenance of teaching practices ill-equipped to address intended curricular expectations. Whether the teacher education programmes, into which the two cohorts above have entered, fulfil such aims, will be the focus of a subsequent paper.

Note

1. This is the literal translation of the Greek expression 'τετράγωνο μυαλό' (pronounced tetragono mialo) used for someone considered mathematically gifted or talented in natural sciences.

References

Adetula, L.O. 1990. Language factor: Does it affect children's performance on word problems? *Educational Studies in Mathematics* 21, no. 4: 351–65.

Aguirre, J., and N.M. Speer. 1999. Examining the relationship between beliefs and goals in teacher practice. *The Journal of Mathematical Behavior* 18, no. 3: 327–56.

Andrews, P. 2007a. Mathematics teacher typologies or nationally located patterns of behaviour. *International Journal of Educational Research* 46, no. 5: 306–18.

Andrews, P. 2007b. The curricular importance of mathematics: A comparison of English and Hungarian teachers' espoused beliefs. *Journal of Curriculum Studies* 39, no. 3: 317–38.

Andrews, P. 2009. Comparative studies of mathematics teachers' observable learning objectives: Validating low inference codes. *Educational Studies in Mathematics* 71, no. 2: 97–122.

Andrews, P. 2011. The cultural location of teachers' mathematical knowledge: Another hidden variable in mathematics education research? In *Mathematical knowledge in teaching*, ed. T. Rowland and K. Ruthven, 99–118. New York: Springer.

Arcavi, A., and A. Friedlander. 2007. Curriculum developers and problem solving: The case of Israeli elementary school projects. *ZDM* 39, no. 5: 355–64.

Bassok, M. 1990. Transfer of domain-specific problem-solving procedures. *Journal of Experimental Psychology: Learning, Memory, and Cognition* 16, no. 3: 522–33.

Beswick, K. 2005. The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal* 17, no. 2: 39.

Blum, W., and M. Niss. 1991. Applied mathematical problem solving, modelling, applications, and links to other subjects – state, trends and issues in mathematics instruction. *Educational Studies in Mathematics* 22, no. 1: 37–68.

Borasi, R. 1986. On the nature of problems. *Educational studies in mathematics* 17, no. 2: 125–41.

Boyatzis, R.E. 1998. *Transforming qualitative information: Thematic analysis and code development*. Thousand Oaks, CA: Sage.

Bransford, J.D., A.L. Brown, and R.R. Cocking. 2000. *How people learn*. Washington, DC: National Academy Press.

Cai, J. 1995. A cognitive analysis of US and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. *Journal for Research in Mathematics Education. Monograph* 7.

Cai, J. 2010. Commentary on problem solving heuristics, affect, and discrete mathematics: A representational discussion. In *Theories of Mathematics Education*, ed. B. Sriraman and L. English, 251–60. Dordrecht: Springer.

Cai, J., and F. Lester. 2005. Solution representations and pedagogical representations in Chinese and US classrooms. *The Journal of Mathematical Behavior* 24, nos. 3–4: 221–37.

Cai, J., and F. Lester. 2007. Contributions from cross-national comparative studies to the internationalization of mathematics education: Studies of Chinese and US classrooms. *Internationalisation and Globalisation in Mathematics and Science Education*: 269–83.

Cai, J., and B. Nie. 2007. Problem solving in Chinese mathematics education: Research and practice. *ZDM* 39, no. 5: 459–73.

Chapman, O. 1997. Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics* 32, no. 3: 201–28.

Cooney, T.J., B.E. Shealy, and B. Arvold. 1998. Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education* 29, no. 3: 306–33.

Cope, C. 2004. Ensuring validity and reliability in phenomenographic research using the analytical framework of a structure of awareness. *Qualitative Research Journal* 42, no. 2: 5–18.

Correa, C.A., M. Perry, L.M. Sims, K.F. Miller, and G. Fang. 2008. Connected and culturally embedded beliefs: Chinese and US teachers talk about how their students best learn mathematics. *Teaching and Teacher Education* 24, no. 1: 140–53.

Cummins, D.D. 1988. The role of understanding in solving word problems. *Cognitive psychology* 20, no. 4: 405–38.

Curricula Development Service of Cyprus – CDS. 1999. *Αναλυτικά Προγράμματα Δημοτικής Εκπαίδευσης* [National Curriculum for Primary Education]. Nicosia: Cyprus Pedagogical Institute.

DfE and QCA. 1999. *The National Curriculum for England: Mathematics.* Her Majesty's Stationery Office.

Drake, C. 2006. Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education* 9, no. 6: 579–608.

Entwistle, N.J., and E.R. Peterson. 2004. Conceptions of learning and knowledge in higher education: Relationships with study behaviour and influences of learning environments. *International Journal of Educational Research* 41, no. 6: 407–28.

Ernest, P. 1989. The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching* 15, no. 1: 13–33.

Fan, L., and Y. Zhu. 2007. From convergence to divergence: The development of mathematical problem solving in research, curriculum, and classroom practice in Singapore. *ZDM* 39, no. 5: 491–501.

Givvin, K.B., J. Hiebert, J.K. Jacobs, H. Hollingsworth, and R. Gallimore. 2005. Are there national patterns of teaching? Evidence from the TIMSS 1999 video study. *Comparative Education Review* 49, no. 3: 311–43.

Goldin, G.A. 2004. Problem solving heuristics, affect, and discrete mathematics. *ZDM* 36, no. 2: 56–60.

Guilford, J.P. 1956. The structure of intellect. *Psychological Bulletin* 53, no. 4: 267–93.

Haggis, T. 2003. Constructing images of ourselves? A critical investigation into 'approaches to learning' research in higher education. *British Educational Research Journal* 29, no. 1: 89–104.

Hofstede, G. 1986. Cultural differences in teaching and learning. *International Journal of Intercultural Relations* 10, no. 3: 301–20.

Jennings, S., and R. Dunne. 1996. A critical appraisal of the National Curriculum by comparison with the French experience. *Teaching mathematics and its applications* 15, no. 2: 49.

Jonassen, D.H. 2003. Designing research-based instruction for story problems. *Educational Psychology Review* 15, no. 3: 267–96.

Kagan, D.M. 1992. Professional growth among preservice and beginning teachers. *Review of Educational Research* 62, no. 2: 129.

Kaiser, G., K. Hino, and C. Knipping. 2006. Proposal for a framework to analyse mathematics education in Eastern and Western traditions. In *Mathematics Education in Different Cultural Traditions – A Comparative Study of East Asia and the West*, 319–51.

Kapa, E. 2001. A metacognitive support during the process of problem solving in a computerized environment. *Educational Studies in Mathematics* 47, no. 3: 317–36.

Kelly, C.A. 2006. Using manipulatives in mathematical problem solving: A performance-based analysis. *The Montana Mathematics Enthusiast* 3, no. 2: 184–93.

Kvale, S., and S. Brinkmann. 2009. *InterViews: Learning the craft of qualitative research interviewing*. London: Sage.

Lester, F.K. 1994. Musings about mathematical problem-solving research: 1970–1994. *Journal for Research in Mathematics Education* 25, no. 6: 660–75.

Marton, F. 1981. Phenomenography-describing conceptions in the world around us. *Instructional science* 10, no. 2: 177–200.

Mason, J., L. Burton, and K. Stacey. 1982. *Thinking mathematically.* Boston, MA: Addison-Wesley.

Muir, T., K. Beswick, and J. Williamson. 2008. "I'm not very good at solving problems": An exploration of students' problem solving behaviours. *Journal of Mathematical Behavior* 27, no. 3: 228–41.

NCTM. 2000. *Principles and standards for school mathematics.* Reston, VA: NCTM.

Nunokawa, K. 1994. Improving diagrams gradually: One approach to using diagrams in problem solving. *For the Learning of Mathematics* 14, no. 1: 34–8.

Nunokawa, K. 1995. Problem solving as modelling: A case of augmented-quotient division problem. *International Journal of Mathematical Education in Science and Technology* 26, no. 5: 721–7.

Nunokawa, K. 2005. Mathematical problem solving and learning mathematics: What we expect students to obtain. *The Journal of Mathematical Behavior* 24, nos. 3–4: 325–40.

OECD. 2001. *Knowledge and skills for life. First results from PISA 2000*. Paris: OECD.

OECD. 2004. *Learning for tomorrow's world. First results from PISA 2003*. Paris: OECD.

OECD. 2007. *PISA 2006 Science competencies for tomorrow's world.* Vol. 1. Paris: OECD.

Op't Eynde, P., E. De Corte, and L. Verschaffel. 2003. Framing students' mathematics-related beliefs. In *Beliefs: A hidden variable in mathematics education?*, ed. G. Leder, E. Pehkonen and G. Törner, 13–37. Dordrecht: Springer.

Op't Eynde, P., E. De Corte, and L. Verschaffel. 2006. "Accepting emotional complexity": A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics* 63, no. 2: 193–207.
Polya, G. 1945. *How to solve it: A new aspect of mathematics.* Princeton, NJ: Princeton University Press.
Santagata, R. 2004. "Are you joking or are you sleeping?" Cultural beliefs and practices in Italian and US teachers' mistake-handling strategies. *Linguistics and Education* 15, nos. 1–2: 141–64.
Schoenfeld, A.H. 1985. *Mathematical problem solving*. Orlando, FL: Academic Press.
Schoenfeld, A.H. 1992. Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. *Handbook of research on mathematics teaching and learning*: 334–70.
Schoenfeld, A.H. 2000. Models of the teaching process. *Journal of Mathematical Behavior* 18, no. 3: 243–61.
Schroeder, T.L., and F.K. Lester. 1989. Developing understanding in mathematics via problem solving. In *New Directions for Elementary School Mathematics*, 31–42. Upper Saddle River, NJ: Prentice Hall.
Silver, E.A., H. Ghousseini, D. Gosen, C. Charalambous, and B.T. Strawhun. 2005. Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *The Journal of Mathematical Behavior* 24, nos. 3–4: 287–301.
Skott, J. 2001. The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education* 4, no. 1: 3–28.
Staub, F.C., and E. Stern. 2002. The nature of teachers' pedagogical content beliefs matters for students' achievement gains: Quasi-experimental evidence from elementary mathematics. *Journal of Educational Psychology* 94, no. 2: 344–55.
Strauss, A., and J. Corbin. 1998. *Basics of qualitative research: Techniques and procedures for developing grounded theory.* London: SAGE Publications.
Swidler, S.A. 2000. Contextual conflicts in educators' personal experience narratives. *International Journal of Qualitative Studies in Education* 13, no. 5: 553–68.
Thompson, A.G. 1984. The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics* 15, no. 2: 105–27.
Van den Heuvel-Panhuizen, M. 2003. The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics* 54, no. 1: 9–35.
Wilson, M., and T. Cooney. 2003. Mathematics teacher change and developments. In *Beliefs: A hidden variable in mathematics education?*, ed. G. Leder, E. Pehkonen and G. Törner, 127–47. Dordrecht: Springer.