

## A Comparison of Numerical Methods and Analytical Methods in Determination of Tunnel Walls Displacement – A Case Study

**Behdeen Oraee-Mirzamani**, Ph.D. Student  
Centre For Environmental Policy  
Imperial College London  
London, United Kingdom

**Saeed Zandi**, Msc Student  
South Tehran Branch  
Azad University  
Tehran, Iran

**Kazem Oraee**, Professor  
Department of Management  
University of Stirling  
Stirling, United Kingdom

### ABSTRACT

Tunnel stability has an important role in the production process of an underground coal mine. There are various methods for analysing tunnel stability, such as numerical methods and analytical methods. In this paper, numerical methods (Phase2 software) have been used to determine tunnel wall displacement in a mining tunnel of the Parvade underground coal mine. The Ground Reaction Curve has also been drawn using analytical methods to determine tunnel wall displacement. The comparison of results from the numerical method and the analytical method show a noteworthy difference in the tunnel wall displacement. The displacement calculated by the numerical method shows a lower value than that of the analytical method because the numerical method is more suited to modeling the various coal and rock layers, as well as the shape of the excavations found in coal mines. The methodology used in this paper together with the results obtained from both methods can serve as useful tools for the coal mine design engineer when determining the ground support requirements.

### INTRODUCTION

Excavation and ore extraction processes cause problems such as a displacement field generated in the orebody and the surrounding rock. This is one of the major engineering problems that engineers are faced with in underground mining (Singh, Singh, and Murthy, 2010). In the design process of an underground coal mine, the stability of tunnels is an important parameter that should be studied carefully because it has an important role in the production process of the mine. Instability and collapse of tunnels may cause different damages. Such damages not only result in an increase in costs, but also are dangerous towards the miners. Therefore, accurate analysis and determination of the displacement in the tunnel's roof and walls can help implement a suitable support system and therefore, make the tunnel more stable.

In this paper, the displacement fields in the roof and walls of tunnels in the Parvade coal mine have been studied. This mine is one of the main coal mines in Iran. Several collapses have occurred in this mine and instability of tunnel's roof and walls is one of the most important problems in this mine.

There are various methods for analysing tunnel stability and the determination of tunnel walls' displacement. Two of the main methods are analytical methods and numerical methods. These methods have been used widely in order to analyse the stability of tunnels during the design process of underground coal mines.

In this research, both methods have been used in order to determine the displacement fields in the case study. A comparison between the two methods was carried out that indicated that there is some difference between the results obtained from the two methods.

After comparing these methods, the suitable method can be chosen based on these results and the mine conditions for the correct calculation of tunnel's roof and walls displacement. The significance of this comes into light when one seeks a suitable implementation method and installation of the support system in the tunnel.

### PARVADE UNDERGROUND COAL MINE

The Tabas coal region is one of the most comprehensive coal resources in Iran. Tabas underground mine is the main mine of the Iranian Coal Enterprises and is located in the Khorasan Province of Iran. The Parvade coalfield lies approximately 85km south of the city of Tabas in mid-eastern Iran. The total probable anthracite reserve in the Parvade region is approximately 1.2 billion tons. Furthermore, the minable reserve suitable for underground production is 28 million tons in the Parvade1 (Manteqi, Shahriar, and Torabi, 2012). Physical and mechanical properties of the rock mass used for numerical modelling are presented in Table1 (IRITEC, 2003; Manteqi, Shahriar, and Torabi, 2012).

### ANALYTICAL METHODS: CONVERGENCE CONFINEMENT METHOD

Analytical methods are very useful in geomechanics because they provide results with very limited effort and highlight the most important variables that determine the solution to problems (Bobet, 2010). In this paper, we used analytical methods (convergence confinement method) to draw the Ground Reaction Curve in order to determine the tunnel walls' displacement in a mining tunnel of the Parvade underground coal mine. The convergence confinement

# 32nd International Conference on Ground Control in Mining

method has been described by other authors (e.g., Gesta et al., 1980; Duncan Fama, Fairhurst, and Carranza-Torres, 2002) and has been commonly used for the support system design in conventional tunnelling. The method achieves a 2D simplified approach for resolving 3D rock support interaction problems.

The main assumption of the convergence confinement method is that the support load required in order to stabilize the excavation decreases with inward tunnel displacement. As the boundary rock moves inward, tangential stresses increase, which results in both yielding of the rock mass and increased confining stresses on the surrounding (Carranza-Torres and Fairhurst, 2000).

## GROUND REACTION CURVE

The Ground Reaction Curve (GRC) can be defined as a curve that describes the decrease of inner pressure and the increase of radial displacement of the tunnel's wall. The GRC analyses the behaviour of the rock mass surrounding the tunnel. Based on the applied pressure on the support system, the GRC determines the tunnel convergence or the displacement of tunnel walls. A typical GRC is shown in Figure 1.

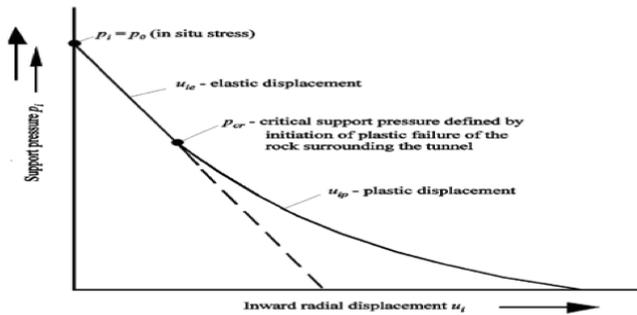


Figure 1. Typical ground reaction curve.

In general, two different types of methods are used to calculate the GRC in a tunnel: analytical methods, which are expressed in mathematical terms and numerical methods, where the differential equations of the problem have to be solved numerically. In this research, the Duncan-Fama analytical method for drawing the ground reaction curve has been used, as implemented in the 'RocSupport' software (RocScience Inc., 2000-2005).

## DUNCAN-FAMA ANALYTICAL METHOD

This method is based on the Mohr-Coulomb failure criterion. Therefore, this method requires parameters such as modulus of elasticity (MPa), Poisson's ratio, internal friction angle, and rock mass compressive strength in order to draw the ground reaction curve (Carranza-Torres and Fairhurst, 1999; 2000). Furthermore, the tunnel was excavated in the coal seam and the weakest member of rocks based on the table 1 for drawing the ground reaction curve has been used. The ground reaction curve for Figure 2 has been drawn using the Duncan-Fama analytical method (RocScience Inc., 2000 – 2005). As evident from Figure 2, the maximum tunnel wall displacement calculated by the Duncan-Fama method is 164mm.

## NUMERICAL MODELLING OPTIONS

Numerical methods are tools that enable engineers to evaluate, both qualitatively and quantitatively, the effects of geology on the design and the consequences of the design on geology. Numerical modelling applications are intended to provide mining and rock mechanics engineers a better understanding for solving problems related to the mine layout and roof support system design to enable consistent and techno-economic viable performance of mining structures throughout their planned life of operations (Singh, Singh, and Murthy, 2010). Numerical methods are promising and effective tools in understanding the rock mass response subjected to complex loading conditions.

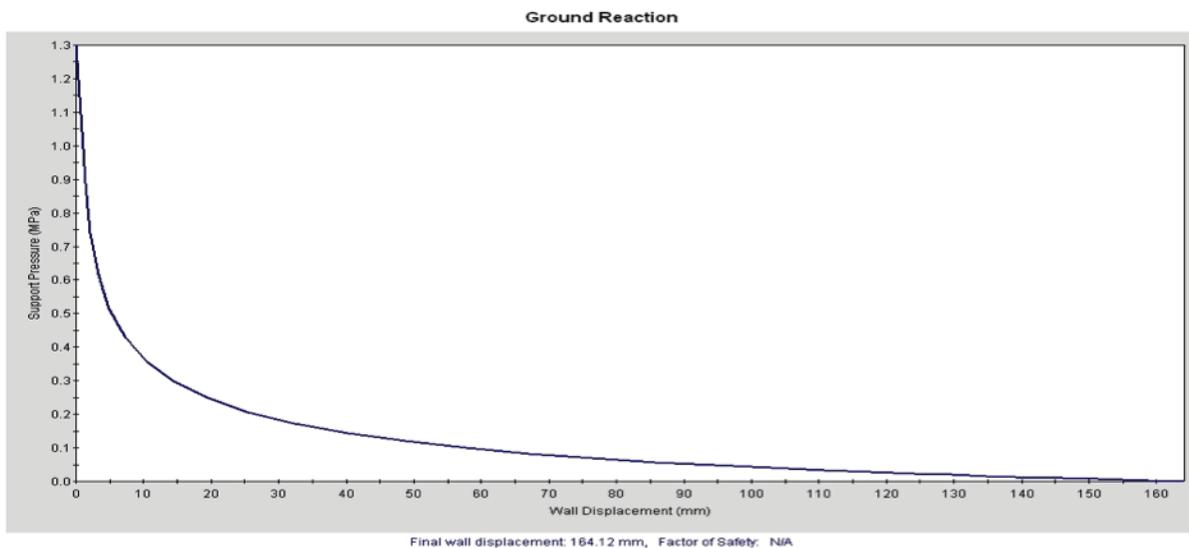


Figure 2. Ground reaction curve drawn using the Duncan-Fama method.

**Table 1. The rock mass input parameters used in numerical modelling (Manteqi, Shahriar, and Torabi, 2012).**

Rock Definition	Siltstone	Sandy Siltstone	Silty Mudstone	Coal	Mudstone	Sandstone
Definition code	1	2	3	4	5	6
Density (M/N/m <sup>3</sup> )	0.0272	0.0271	0.0268	0.016	0.0263	0.027
Internal Friction angle( $\varphi$ )	27.42	31.75	22.17	15.76	20.13	43.52
Cohesion c (MPa)	0.357	0.443	0.257	0.084	0.231	0.767
Modulus of elasticity E (MPa)	2238	2818	1778	749	1995	3548
Tensile strength (MPa)	0.012	0.007	0.005	0.002	0.013	0.017
Poisson's ratio $\nu$	0.25	0.25	0.28	0.25	0.31	0.25
Bulk modulus <sup>a</sup> (K) (MPa)	1492	1878	1347	499	1750	2365
Shear modulus <sup>b</sup> (G) (MPa)	895	1127	695	299	761	1419
Uniaxial compressive strength (MPa)	0.273	0.287	0.114	0.015	0.165	1.01
<sup>a</sup> (K) = $E/3(1-2\nu)$ <sup>b</sup> (G) = $E/2(1+\nu)$						

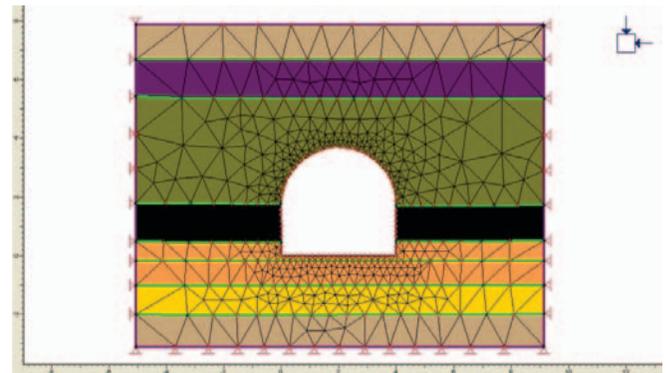
The numerical methods for rock mechanics problems are divided into two main groups: the continuum methods and the discrete methods. Although the choice of continuum or discrete methods mainly depends on the problem scale and fracture system geometry, the choice also depends on other problem-specific factors (Jing and Hudson, 2002).

For the continuum methods, there are three main approaches: the Finite Difference Method (FDM), the Finite Element Method (FEM), and the Boundary Element Method (BEM). The main discrete methods include the Discrete Element Methods (DEM) and the Discrete Fracture Network (DFN) methods (Jing and Hudson, 2002). It is important to be aware that the drawback of analytical methods is that they cannot represent complex in-situ conditions and geometry, and this is the main reason that the numerical methods are used. The numerical methods should be used carefully and with a complete understanding of the background of the problem that is being analysed in order to obtain accurate conclusions.

### FINITE ELEMENT METHOD

The finite element method is perhaps the most widely applied numerical method across the science and engineering fields. Since its origin in the early 1960s, much finite element method development work has been specifically oriented towards rock mechanics problems (Jing and Hudson, 2002). This is because it was the first numerical method with enough flexibility for the treatment of material heterogeneity, non-linear deformability, complex boundary conditions, in-situ stresses, and gravity.

In this paper, we used the Phase2 software to determine the tunnel walls' displacement in this particular case study. Phase2 is a two-dimensional finite element program that calculates stresses and displacements around underground openings. It can be used



**Figure 3. The created model in Phase2.**

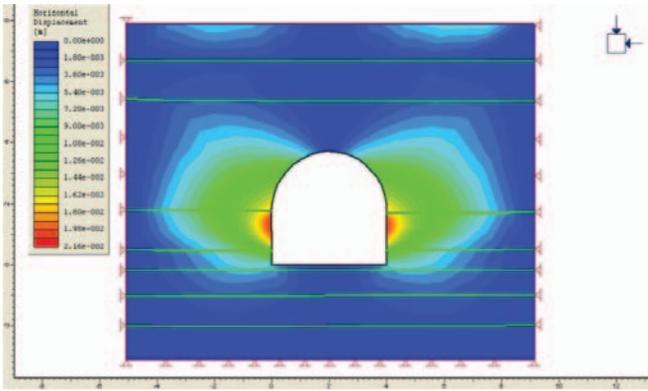
to solve a wide range of mining and civil engineering problems (Roccience, Inc., 2002).

### MODELLING IN PHASE2

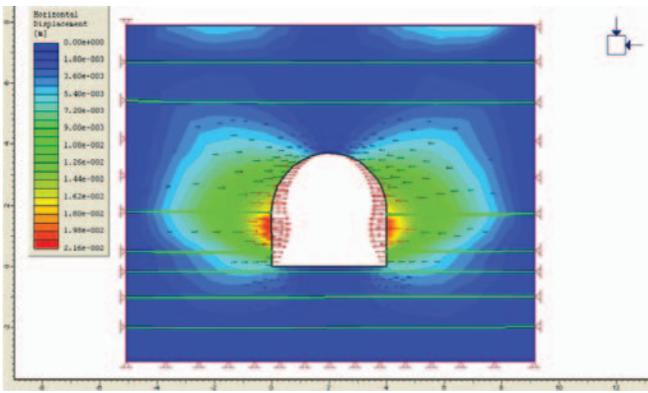
The first step of numerical analysis with the Phase2 software is modelling the underground opening in the computer. In this stage, we enter and edit the model boundaries, in situ stresses, and material properties and create the finite element meshes. The created model in Phase2 is shown in Figure 3.

The extracted results of the software are shown in Figure 4. As it can be seen in Figure 4, the horizontal displacement on the tunnel walls is high. Furthermore, Figure 4 shows the deformation vectors on the tunnel walls.

According to the extracted results from Phase2, the maximum tunnel wall displacement is 20mm (Figure 5).



**Figure 4. Horizontal displacement around the tunnel (Note: on the tunnel walls).**



**Figure 5. Maximum tunnel wall displacement.**

The displacement of the tunnel walls calculated by the numerical method (Phase2 software) shows a lower value than that of analytical method (Duncan-Fama method).

## CONCLUSIONS

In this paper, the displacement field was calculated for the mining tunnel's roof and walls of Parvade coal mine. It is necessary to determine the displacement in the tunnel's roof and walls after excavation so that suitable implementation and installation of support systems take place in the tunnel due to the existence of historical collapses in the Parvade mine. The displacement field was calculated using both analytical and numerical methods.

After comparing the results obtained from each method, it was found that the numerical method shows a lower value of displacement in the tunnel walls in comparison with the analytical method. There was some difference between these methods' results owing to the fact that their capabilities and assumptions are different to one another. The analytical method requires several simplifying assumptions, which contributes to large displacements being predicted. Firstly it assumes that the excavation is circular in a uniform stress field, which is seldom true in coal mining. In addition, the method can only consider a single material type. In this study, it was assumed the entire rock mass consists of coal, the weakest material. This simplifying assumption led to the very large predicted displacements. On the other hand, the finite element model was able to accurately simulate the various rock types, field

stress conditions and shape of the excavation, producing more realistic displacement results. After this comparison, the finite element method appears to be a more suitable method for stability analysis of tunnels in coal mines (based on these results, the mine observed conditions and engineering judgment). Therefore, special care has to be taken when using each of these methods since their assumptions and limitations may have a great effect on the final result. Furthermore, one of the objectives of this research was to show these different assumptions and limitations and their effect on the results.

Analytical solutions, often have limited application because they must be used within the range of assumptions made for their development. Such assumptions usually include elastic behaviour; homogeneous, isotropic material; time independent behaviour; quasi-static loading, and the ratio of horizontal stress to vertical stress being constant (it is often equal to 1). Rocks and soils may not be isotropic or homogeneous and the loading may not be static. Additionally, the geometry of the problem may be complex. In these cases, solutions can only be obtained numerically. One of the other limiting assumptions in drawing the ground reaction curve (and determining the tunnel walls' displacement) is the circular cross section of the tunnel. Numerical methods can widely be used in order to perform stability analysis in every underground excavation with different shapes and dimensions without the analytical method's limited assumptions.

## REFERENCES

- Bobet, A. (2010). "Numerical methods in geomechanics." *The Arabian Journal for Science and Engineering*.35(1B).
- Carranza-Torres, C., Fairhurst, C. (1999). "The elasto-plastic response of underground excavations in rock masses that satisfy the Hoek-Brown failure criterion." *International Journal of Rock Mechanics and Mining Science*.36:777-809.
- Carranza-Torres, C., Fairhurst, C. (2000). "Application of the convergence-confinement method of tunnel design to rock masses that satisfy the Hoek-Brown failure criterion." *Tunneling and Underground Space Technology*. 15(2):187-213.
- Duncan Fama, M.E. (1993). "Numerical Modelling of Yield Zones in Weak Rocks" In: Hudson JA (ed) *Comprehensive Rock Engineering*, 2. Pergamon, Oxford, pp. 49-75
- IRITEC. (2003). *Tabas Coal Mine Project, Detailed Engineering Report*, vol. 1, Underground Mine Revision A, IRITEC.
- Jing, L., Hudson, J.A. (2002). "Numerical methods in rock mechanics." *International Journal of Rock Mechanics and Mining Science*.39:409-427.
- Manteqi, H., Shahriar, K., Torabi, R. (2012). "Prediction of first weighting distance in longwall coal mining by 3D numerical modeling – A case study." In: *Proceedings of the 31<sup>st</sup> International Conference on Ground Control in Mining*.
- Rock Science Inc. (2000 – 2005). *RocSupport (Rock support interaction and deformation analysis for tunnels in weak rock), Tutorial Manual*, Canada: Rock Science Inc.

## 32nd International Conference on Ground Control in Mining

Rock Science Inc. (2002). Phase2 User's Guide, finite element analysis and support design for excavations. Canada: Rock Science Inc., pp.54–72.

Singh, G.S.P, Singh, U.K, Murthy, V.M.S. (2010). "Application of numerical modeling for strata control in mines." *Geotechnical and Geological Engineering*. 28:513–524.