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Can Common Stocks Provide A Hedge Against Inflation? Evidence from African Countries

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Abstract

The extent to which the stock market provides a hedge to investors against inflation is examined for African stock markets. By employing parametric and nonparametric cointegration procedures, we show that the point estimates of the elasticities of stock prices with respect to consumer prices range from 0.015 for Tunisia to 2.264 for South Africa, evidence of a positive long-run relationship. Further, the time path of the response of stock prices to innovations in consumer prices exhibits a transitory negative response for Egypt and South Africa, which becomes positive over longer horizons: important indication that the stock market tends to provide a hedge against rising consumer prices in African markets.

Keywords: Stock Prices, Inflation, Fisher Effect, African Stock Markets, Cointegration

JEL: G10, G15, C32

1. Introduction

The relationship between interest rates and inflation has been investigated in both theoretical and empirical economics. If the ex ante real rate of interest is assumed constant, economic agents will require a nominal return that will compensate for the marginal utility of forgone current consumption (measured by the real interest rate) and the decline in the purchasing power of money. This proposition implies that nominal interest rates move one-for-one with inflation, hence a permanent change in the rate of inflation has no long-run effect on the level of the real interest rate. This relationship is typically referred to as the Fisher hypothesis—formalized in Fisher (1930). Transposing this notion to stock markets implies a positive, one-to-one relationship between stock returns and inflation (see Anari and Kolari, 2001). Thus, in a competitive market stock returns may serve as a hedge against inflation.

However, a large body of evidence indicates that the stock market tends to perform poorly during inflationary periods (Barnes et al, 1999). Spurred by rising inflation in the 1970s, Bodie (1976), Nelson (1976) and Fama and Schwartz (1977) compared the inflation hedge properties of common stocks with those of other financial and real variables for the US and found that common stocks were a poor hedge not only against unexpected inflation, but also against expected inflation¹. Gultekin (1983) in a study of 26 countries during the post war period consistently failed to find support for the hypothesis that common stocks and the expected inflation rate were independent. A number of arguments have been put forward for the observed relationship between stock returns and inflation, i) the inflation illusion hypotheses by Modigliani and Cohn (1979) which argues that investors undervalued stocks in the 1970's because they used nominal interest rates to discount cash flows and also excluded capital gains that accrued to firms with fixed rate debt liabilities; ii) Feldstein (1980) real after-tax hypothesis which posits corporate profits vary inversely with inflation as a result of higher effective tax

¹ Notable exceptions are Firth (1979), and Gultekin (1983) who find reverse evidence for the UK and long horizon studies such as Boudoukh and Richardson (1993).

rates due to higher inflation; iii) Fama (1981) proxy hypothesis that holds that an inverse relationship between real stock returns and inflation is spurious because inflation acts as a proxy for real-activity variables in models that relate stock returns to inflation; iv) the risk-premium hypothesis by Devereux and Yetman (2002), and more recently v) Anari and Kolari (2010) show through simulation that nominal discount rates can have a negative impact on stock values in the short-run due to inflation premium included in the discount rate. Geske and Roll (1983) present reverse causality arguments and Kaul (1987, 1990) argue for the effects of monetary developments on inflation.

Most of the evidence regarding the relationship between the stock market and inflation are derived from regressions of real ex-post stock returns on expected inflation or unexpected inflation over short periods (for the limitations of this approach see Gallagher, 1986). As Hendry (1986) and Juselius (1991) observe, when a time series is differenced long-run information contained in the levels of the variable is lost.

Since the 1990s, the Fisher hypothesis has undergone empirical tests that take the potential nonstationarity and cointegration properties of the involved series explicitly into account (see Mishkin, 1992)². International evidence by Ely and Robinson (1997) shows that stocks do maintain their value relative to movements in overall consumer price indices and this is invariant to the source of inflation. Anari and Kolari (2001) also employed a cointegration approach with data from 6 industrialized countries (US, UK, Canada, France, Germany, and

²For the relationship between interest rates and inflation, see Evans and Lewis (1995), Crowder and Hoffman (1996) and Koustas and Serletis (1999). Evans and Lewis (1995), for instance find evidence in favour of a long-run Fisher effect, with coefficient of less than unity for the US, whereas Crowder and Hoffman (1996) find coefficient estimates greater than unity, evidence consistent with tax augmented versions of the Fisher hypothesis.

Japan). They show that the long-run generalised³ Fisher elasticities of stock prices with respect to consumer prices exceed unity and are in the range of 1.04 to 1.65, which tend to support the Fisher effect. In the same spirit, Luintel and Paudyal (2006) analysed whether aggregate and disaggregate industry indices in the UK provide a hedge against inflation. Thus studies based on long-run relationships tend to be more supportive of the Fisher type explanation than static short-run estimates.

In recent time, the literature on the relationship between inflation and stock returns has shifted to examining the nature of the shock in different economic states. While the predicted relationship is still a subject of debate, depending on the data set characteristics (i.e. whether the measure of inflation is based on PPI or CPI, and the frequency of the series used), the country or the econometric methodology employed, McQueen and Roley (1993) find significant and negative one day stock market responses to CPI inflation shocks in medium economic states but not in high and low economic states. In addition, one day PPI inflation shocks are significant in high economic state but not in other states. Knif et al (2008) employ event studies by modifying the technique to account for macroeconomic announcements when measuring the cumulative effect on stock returns. They test the hypothesis that (a) positive inflation shocks in good (bad) times are perceived by stock investors as bad (good) news and (b) negative inflation shocks in good (bad) economic times are good (bad) news. The authors show that when negative and positive shocks are pooled across economic states, their effects on aggregate stock returns are washed out or muted. This finding reconcile the apparent disparity between regression results, that find an inverse stock return and inflation relation and event studies reporting a weak or insignificant relation. The empirical validity of the

³ We refer to the relationship between stock prices and consumer prices as the generalized Fisher effect.

generalized Fisher hypothesis has also profound implications on investment (see Fama and Gibbons, 1982, and Shrestha et al, 2002)

Following the economic restructuring in the 1980s and the financial reforms that ensued, most African countries have generally adhered to strict monetary and fiscal policies. In spite of these efforts, however, inflation in African countries has assumed a general upward trend. Annual inflation in the sampled countries (Egypt, Kenya, Morocco, Nigeria, South Africa and Tunisia) has averaged 10.8% (i.e. between 1990 and 2004); with some countries, experiencing rates in excess of 30% (Nigeria and Kenya in the early to mid-1990s)⁴. Inflation of this magnitude have significant adverse effects on the financial sectors of African countries, particularly in the context of fixed nominal interest rates, the choice of investment vehicles, and the composition of individual basket of assets. Two crucial questions that have not been addressed is whether stock markets in African countries offer a shelter to investors in the face of rising inflation, and how do stocks perform under inflationary conditions?

This paper makes two key contributions: First, there has been relatively limited work in the literature testing the validity of the Fisher hypothesis with respect to stock markets in Africa—the markets have grown increasingly important⁵ as avenues for global portfolio diversification (see Harvey, 1995). Secondly, the paper investigates countries that have adopted inflation targeting (South Africa) and countries with high inflation rates. From an econometric point of view we i) provide recursive estimation for both the OLS estimates and the trace test (eigenvalues), ii) employ both parametric and nonparametric cointegration techniques and iii) impulse response functions are presented with bootstrapped standard errors.

⁴The most commonly cited reason for high inflation in African countries is money growth, although, exchange rate depreciation is also important.

⁵ “Buy Africa” Economist 19/2/2008.

The remainder of the paper is organised as follows: the next section presents the model. Section 3 presents the data and section 4 looks at the long-run relationship between stock prices and goods prices. The last section concludes.

2. The Theoretical Model

The Fisher equation encapsulates the relationship that exists between nominal interest rates and expected inflation. If the ex ante real rate of interest is assumed constant, then economic agents will require a nominal return that will compensate for the marginal utility of the forgone current consumption (measured by the real interest rate) and the decline in the purchasing power of money. The decline in the purchasing power of money is commonly proxied by the rate of price inflation that is expected to occur over the life of the loan. Therefore, the Fisher equation is given in its most simple form as

$$R_t = (E_{t-1}[r_t]) + (E_{t-1}[\pi_t]) + u_t \quad (1)$$

where R_t is the nominal interest rate, $(E_{t-1}[r_t])$ and $(E_{t-1}[\pi_t])$ are the ex-ante real interest rate and the expected inflation (defined as $\pi_t = P_t - P_{t-1}$), respectively, and $E[\bullet]$ denotes the conditional expectation operator. Imposing rational expectations, the expected and the actual inflation rate may differ by a stationary zero mean forecast error v_{1t} , obtaining

$$\pi_t = E_{t-1}[\pi_t] + v_{1t} \quad (2)$$

and, similarly, the ex-post real interest rate is the sum of the ex-ante real rate and a forecast error v_{2t} , such that

$$r_t = E_{t-1}[r_t] + v_{2t} \quad (3)$$

The inflation rate and nominal interest rate are observable and the ex-post real interest rate is

$$r_t = R_t - \pi_t + v_t^{(1)} \quad (4)$$

where $v_t^{(1)} = u_t - v_{1t} - v_{2t}$.

Equation (4) provides the basis for testing the Fisher hypothesis (see Rapach and Weber, 2004). Assuming $v_t^{(1)}$ to be stationary, the integration properties of r_t are determined by the integration properties of R_t and π_t . If the latter variables are both stationary ($R_t, \pi_t \sim I(0)$) then $r_t \sim I(0)$. However, if both variables are nonstationary i.e. ($R_t, \pi_t \sim I(1)$), then there may exist a cointegrating relationship between interest rates and inflation with cointegrating vector $(1, -1)$.

2.1. The Empirical Model

In the context of stock markets, the Fisher hypothesis postulates that the nominal stock return reflects market expectations about the real stock return and inflation; a 1% increase in expected inflation should be associated with a 1% increase in stock returns. Thus, investment in stocks may be used as a complete hedge against inflation. Before we examine the long-run relationships, we start with the regression of stock returns on contemporaneous inflation:

$$\Delta S_t = \alpha + \beta E(\Delta P_t | \varphi_{t-1}) + \varepsilon_t \quad (5)$$

where ΔS_t and $\Delta P_t = \pi_t$ are the nominal stock return and inflation from $t-1$ to t respectively; α is the expected real rate of stock returns; $\beta = 1$ if the Fisher hypothesis holds;

$E(\Delta P_t | \varphi_{t-1})$ is the expectation of inflation based on the information set φ_{t-1} available at $t-1$

and finally ε_t is the error term. Because expected inflation is not available in general, estimation of (5) has to rely on a regression of observables such as

$$\Delta S_t = \alpha + \beta \Delta P_t + u_t \quad (6)$$

with u_t as a residual. A unit coefficient, $\beta = 1$, would imply that common stocks are a hedge against inflation. However, when the income from stocks is subject to taxes, the rate of return

on common stocks should exceed the inflation rate at least by the tax rate. Therefore, the size of the coefficient (β) could, in fact, exceed unity.

Using stock returns and inflation tell us only about the short-run solution. To investigate the long-run relationship between stock prices (S_t) and consumer prices (P_t), we apply Johansen's (1995) multivariate method (where stock prices and goods prices are defined as the stock price index and consumer price index respectively). Under this approach, a system of endogenous variables can be parameterized on a vector error correction model (VECM):

$$\Delta y_t = \mu + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \dots + \Gamma_{k-1} \Delta y_{t-k+1} + \Pi y_{t-k} + e_t \quad (7)$$

where $y_t = (S_t, P_t)'$ collects observations of stock prices and consumer prices in each country; $e_t \sim iid(0, \Sigma)$; μ is a (2×1) vector of intercepts; Γ and Π are (2×2) coefficient matrices. If y_t is integrated of order one, and cointegrated with cointegration rank $r=1$, the matrix Π allows a factorization as $\Pi = \alpha\beta'$, where both α and β are 2×1 vectors. To test for cointegration we look at the rank of the Π matrix via its eigenvalues. Since the rank of a matrix is the number of non-zero eigenvalues (λ), the number of $\lambda > 0$ represents the number of cointegrating vectors among the variables. The test for non-zero eigenvalues is conducted using the trace statistic:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^g \ln(1 - \hat{\lambda}_i) \quad (8)$$

where $\hat{\lambda}_i$ is the estimated eigenvalue and T is the number of observations. If there is cointegration we can then write $(S_{t-1} - \gamma - \theta P_{t-1})$ as the vector of deviations from the long-run relation between S_t and P_t , and can be normalized and expressed as

$$S_t = \gamma + \theta P_t \quad (9)$$

Given that the variables are expressed in logarithms, the coefficient θ is the elasticity of stock prices with respect to consumer prices. Possible outcomes include $\theta > 0$ partial hedge, $\theta = 1$ one-to-one relationship, full hedge and $\theta > 1$ stock returns performance superior.

3. Data and Preliminary Evidence

The data set consists of monthly stock price indices and consumer price indices from 6 African countries (see Table 1 for sample period and basic statistics). We employ All Share indices from the International Financial Statistics (IFS database) of the International Monetary Fund (IMF) for the following countries: South Africa, Nigeria and Kenya. The stock indices are composed of the most actively traded stocks in each country and include at least 70% of the value of shares traded.

For the goods prices we utilize the monthly consumer price index (CPI) for each country as reported by the IMF (IFS database). Indices for consumer prices are the most frequently used indicators of inflation and reflect changes in the cost of acquiring a fixed basket of goods and services by the average consumer. Preference in the IMF calculation is given to series having wider geographical coverage and relating to all income groups.

We use monthly data because long annual series are not available for most of the countries over a sufficiently long time. We begin with a brief descriptive analysis contrasting average inflation rates with stock returns. Monthly inflation is calculated from the individual countries consumer price indices (P_t) as $\Delta P_t = 100 (\ln P_t - \ln P_{t-12})$ and monthly stock returns as $\Delta S_t = 100 (\ln S_t - \ln S_{t-12})$. The monthly (unannualised) stock returns and inflation is shown in Table 1 with the corresponding graphs in Figures 1a to 1f.

Table 1: Descriptive Statistics

| | Inflation | | | | Stock returns | | | | Sample period |
|---------------------|-----------|----------|----------|----------|---------------|----------|----------|----------|--------------------|
| | μ | σ | Skewness | Kurtosis | μ | σ | Skewness | Kurtosis | |
| Egypt | 0.247 | 0.286 | 2.12 | 8.68 | 0.064 | 4.77 | 0.422 | 5.96 | 1990M02 to 2006M12 |
| Kenya | 1.068 | 1.91 | 1.74 | 8.35 | 1.397 | 10.5 | 4.229 | 36.5 | 1990M02 to 2006M12 |
| Morocco | 0.163 | 0.76 | 0.58 | 4.99 | 0.979 | 5.19 | 0.172 | 5.22 | 1995M02 to 2006M12 |
| Nigeria | 1.790 | 2.57 | 0.53 | 3.29 | 2.732 | 6.45 | 0.779 | 6.49 | 1991M08 to 2006M10 |
| South Africa | 0.813 | 0.63 | 0.68 | 4.37 | 1.101 | 5.31 | -0.735 | 5.16 | 1980M02 to 2007M01 |
| Tunisia | 0.247 | 0.27 | 0.28 | 4.14 | 0.064 | 4.77 | 1.071 | 9.45 | 1996M01 to 2006M10 |

Notes: μ , σ are the means and standard deviations respectively.

Average monthly inflation range from 0.16% for Morocco to 0.81% for South Africa. The corresponding stock returns range from 0.98% per month for Morocco to 1.1% per month for South Africa. In Kenya and Nigeria, inflation is typically on the ascendancy especially up to the mid 1990s, which can be attributed mainly to money growth. On an annualized basis, inflation for Nigeria over the sample period is 21.5%; that of Kenya is 12.7%. The return on the NSE All Share Index and the NSE20 share price index over the same period was 32.8% and 16.8% for Nigeria and Kenya respectively.

Figure 1: Monthly stock returns and inflation in African countries

Figure 1a: Egypt

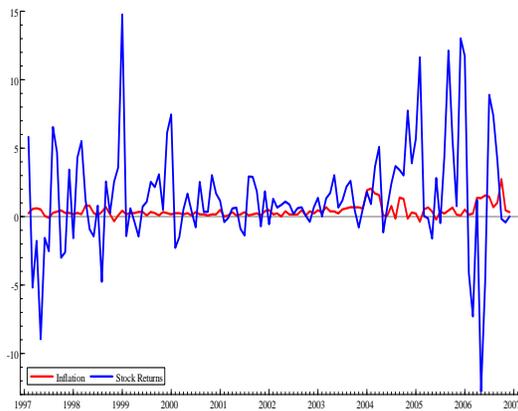


Figure 1b: Kenya

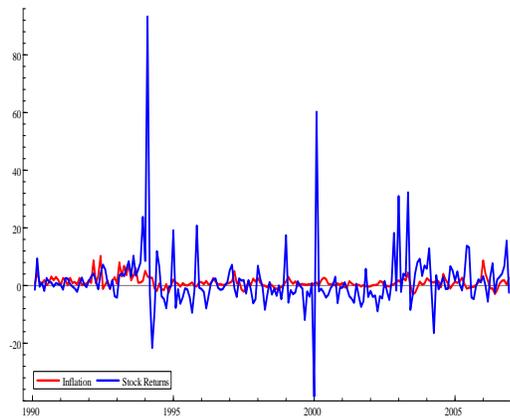


Figure 1c: Morocco

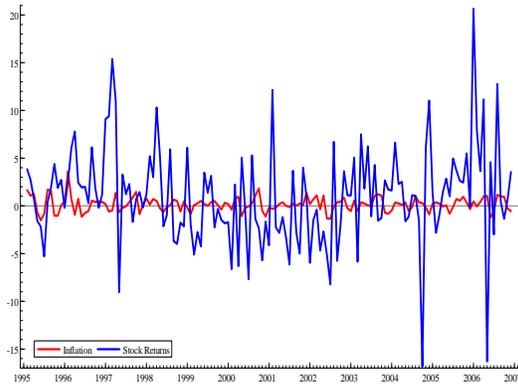


Figure 1d: Nigeria

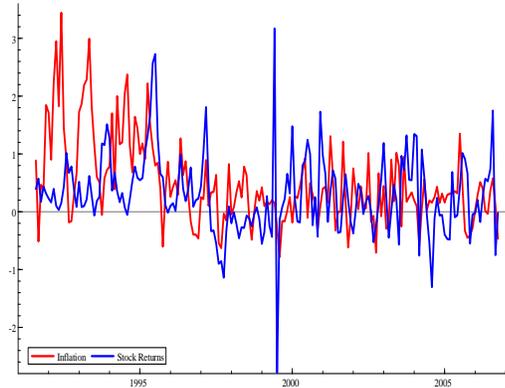


Figure 1e: South Africa

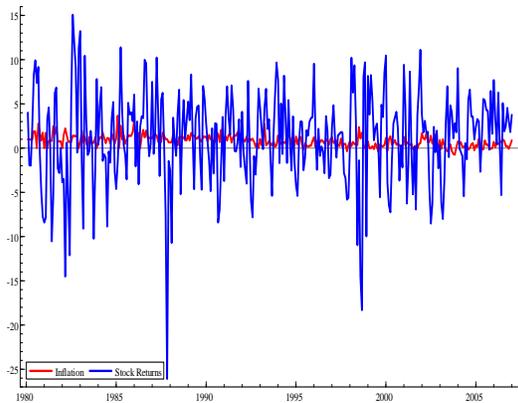
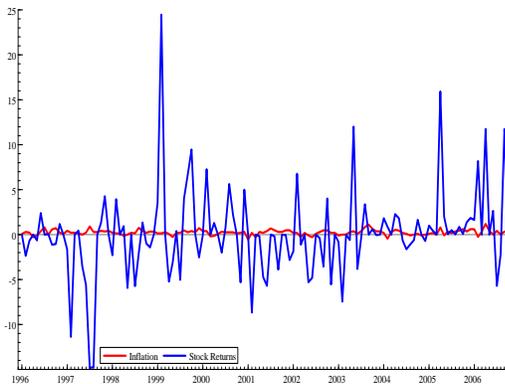


Figure 1f: Tunisia

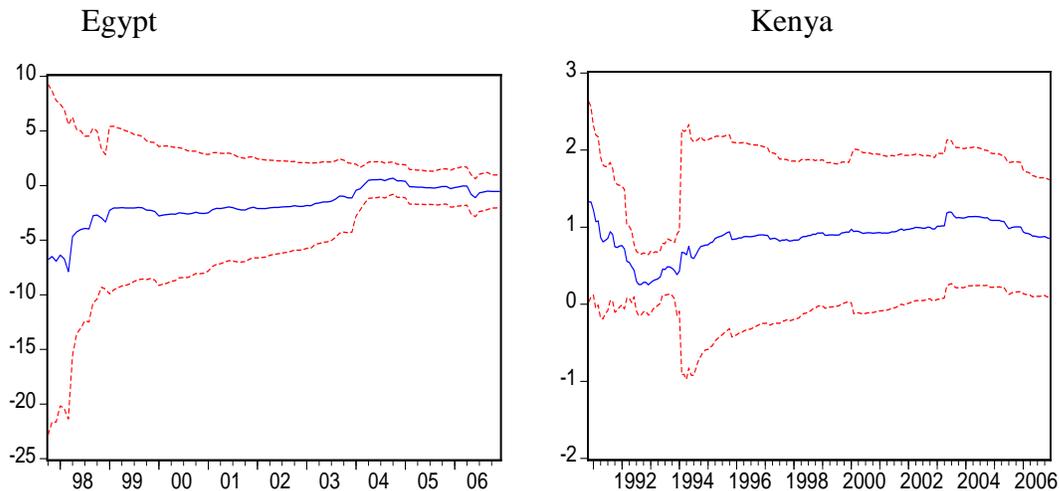


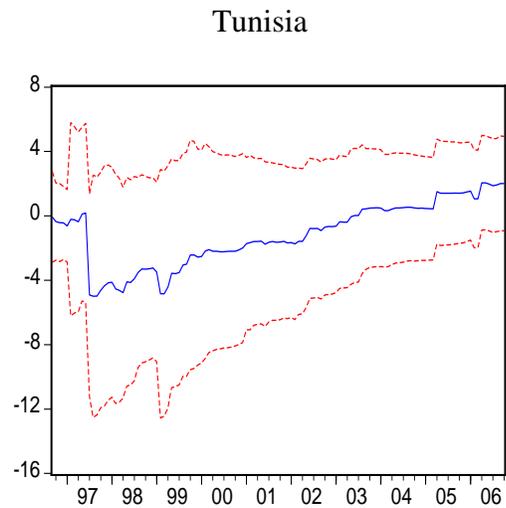
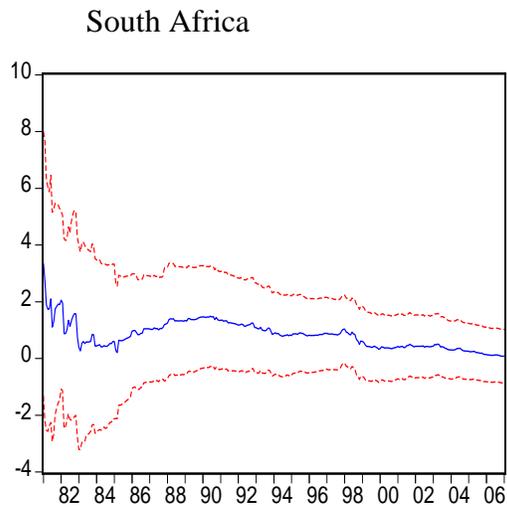
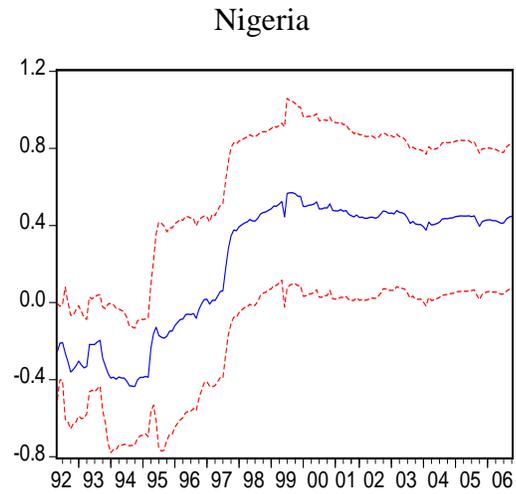
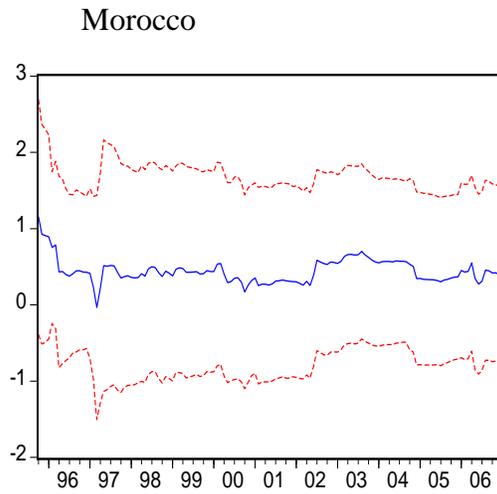
The graphical representation of the series appears in Figure 1. However, as Table 1 and Figure 1 indicate, not only is the mean of inflation and stock returns important, but also their variability (as measured by the standard deviation σ). Monthly inflation has been very volatile especially for Nigeria (2.6%) (see Figure 1d). Lastly, both stock returns and inflation shows excess kurtosis. With the exception of stock returns for South Africa, both series are positively skewed in all countries and the distributional characteristics of the two series appear to be inconsistent with the normality assumption (the leptokurtic characteristics of the data do not affect cointegration analysis, see Rahbek et al, 2002).

3.1. The Contemporaneous Relationship between Stock Returns and Inflation

As a precursor to the long-run analysis, we estimate the relationship between inflation and stock returns. Since the examination of the relationship between inflation and stock returns has not been studied (to the best of our knowledge) in African countries, this seems to be a useful starting point. We regress stock returns on contemporaneous inflation (see appendix). Recursive OLS estimates of the beta coefficients are presented in Figure 2. Of the six countries in our sample, only one country (Egypt) has negative slope estimate (albeit insignificant). This is in contrast to worldwide evidence of a negative relationship between stock returns and inflation. Of the remaining five countries, the relationship between stock returns and inflation is only significant for Kenya and Nigeria (both at the 5% level).

Figure 2: Recursive estimation of beta (± 2 S.E)





The generalized Fisher hypothesis (the null of $\beta = 1$) is not rejected by the Wald test only for Kenya (see also the appendix). The model fits the data very poorly as in all cases we observe low R^2 's. There is also evidence of serial correlation for South Africa, Nigeria, and Egypt as indicated by both the Breusch-Godfrey and Durbin-Watson test for higher order and first order residual correlations respectively. In general, the results appear to be influenced by the high variance of stock returns and, therefore, these estimates must be interpreted with caution. Moreover, using the variables in their first-differences may throw away significant information

about their long-run relationships (see Hendry, 1986 and Juselius, 1991). In the sections that follow, we use the levels of the consumer price and stock price indices to analyse the long-run relationships.

4. Long-run relationship between consumer prices and stock prices

4.1. Unit roots and Stationarity Tests

The long-run relationship between stock prices (S_t) and consumer prices (P_t) crucially depend on the integration and stationarity properties of the two series. We employ two unit root tests; the Philips-Perron (PP), and Breitung (2002) nonparametric test, and the stationarity test suggested by Kwiatkowski et al (1992) i.e. KPSS. The KPSS tests the null of stationarity, whereas PP and Breitung test the null of unit root. The results are shown in Table 2. The results are presented for two scenarios; trend and no trend: ξ_μ and ξ_τ for PP, η_μ and η_τ for KPSS, and τ_μ and τ_τ for Breitung, respectively. As indicated by the second and sixth column of Table 2 we cannot reject the null of a unit root for the levels of both goods prices and stock prices in all countries except Morocco. The PP test indicates that consumer prices are I(0) in Morocco. However, the PP test may fail to reject the null frequently because of low power (KPSS, 1992).

By testing for both the unit root hypothesis and the stationarity hypothesis, one can distinguish series that appear to be stationary, series that appear to be integrated, and series that are not very informative about whether or not they are stationary or have a unit root.

Table 2: Unit Root and Stationarity test

| | No Trend | | | | Trend | | | | No Trend | | Trend | |
|--------------|----------------|-----------------|-------------------|--------------------|-----------------|-----------------|--------------------|--------------------|-----------------|--------------------|------------------|------------------|
| | PP | | KPSS | | PP | | KPSS | | Breitung | | | |
| | Levels | Diff | Levels | Diff | Levels | Diff | Levels | Diff | Levels | Diff | Levels | Diff |
| | PP_{ξ_μ} | PP_{ξ_τ} | $KPSS_{\eta_\mu}$ | $KPSS_{\eta_\tau}$ | PP_{ξ_τ} | PP_{ξ_τ} | $KPSS_{\eta_\tau}$ | $KPSS_{\eta_\tau}$ | BT_{τ_μ} | BT_{τ_μ} | BT_{τ_τ} | BT_{τ_τ} |
| Egypt | | | | | | | | | | | | |
| P_t | 1.293 | -8.357** | 1.116 | 0.216** | -1.415 | -8.38** | 0.173 | 0.073** | 0.094 | 0.015 | 0.021 | 0.002* |
| S_t | -2.021 | -9.732** | 0.980 | 0.383** | -1.120 | -9.82** | 0.139 | 0.076** | 0.093 | 0.003** | 0.015 | 0.000** |
| Kenya | | | | | | | | | | | | |
| P_t | 1.226 | -7.475** | 1.617 | 0.246** | -0.955 | -7.47** | 0.167 | 0.119** | 0.089 | 0.007** | 0.018 | 0.001 |
| S_t | -0.894 | -13.65** | 2.401 | 0.181** | -1.320 | -13.65** | 0.192 | 0.159* | 0.026 | 0.001 ^a | 0.012 | 0.001* |
| Morocco | | | | | | | | | | | | |
| P_t | -0.605 | -11.62** | 1.219 | 0.080** | -4.646** | -11.59** | 0.085** | 0.080** | 0.096 | 0.002 | 0.001 | 0.000** |
| S_t | -0.987 | -12.51** | 0.226** | 0.201** | -1.211 | -12.48** | 0.150* | 0.193* | 0.019 | 0.019 | 0.012 | 0.002* |
| Nigeria | | | | | | | | | | | | |
| P_t | 1.884 | -9.149** | 1.446 | 0.385** | -0.901 | -9.125** | 0.254 | 0.095** | 0.087 | 0.016 | 0.019 | 0.002* |
| S_t | 2.368 | -11.35** | 1.290 | 0.487* | -0.027 | -11.51** | 0.325 | 0.076** | 0.091 | 0.001 | 0.011 | 0.001* |
| South Africa | | | | | | | | | | | | |
| P_t | 2.642 | -13.82** | 2.593 | 0.109** | -2.987 | -13.98** | 0.506 | 0.149* | 0.097 | 0.026 | 0.025 | 0.001** |
| S_t | 4.868 | -13.04** | 2.203 | 0.666* | 3.689 | -13.12** | 0.352 | 0.194* | 0.096 | 0.00** | 0.003 | 0.000** |
| Tunisia | | | | | | | | | | | | |
| P_t | 1.293 | -8.35** | 1.116 | 0.216** | -1.415 | -8.38** | 0.173* | 0.073** | 0.098 | 0.001** | 0.005 | 0.001** |
| S_t | -2.021 | -9.73** | 1.249 | 0.604* | -1.120 | -9.82** | 0.603 | 0.086** | 0.017 | 0.004** | 0.009 | 0.001** |

Note: PP=Philip-Perron; P_t is consumer prices and S_t is the stock index. PP bandwidth selection based on Newey-West. ξ_μ and ξ_τ ; η_μ and η_τ ; τ_μ and τ_τ denotes a constant and constant with linear time trend in the PP, KPSS and Breitung tests respectively

Critical values of PP taken from MacKinnon (1991); KPSS from Kwiatkowski *et al* (1992) and Breitung (2002)

| PP_{ξ_μ} | PP_{ξ_τ} | $KPSS_{\eta_\mu}$ | $KPSS_{\eta_\tau}$ | BT_{τ_μ} | BT_{τ_τ} |
|----------------|-----------------|-------------------|--------------------|-----------------|------------------|
| 1% -3.43 | -3.93 | 0.739 | 0.216 | | |
| 5% -2.86 | -3.41 | 0.463 | 0.146 | 0.01004 | 0.01781 |
| 10% | | | | 0.01435 | 0.00438 |

**significance at the 1% level

* Significance at the 5% level

^a Significance at the 10% level

The KPSS for level and trend stationarity are also presented in columns 4 and 5 and columns 8 and 9 of Table 2. Using the 5% conventional level of significance, the KPSS rejects the null of stationarity in consumer and stock prices for all countries except Morocco (both the KPSS and

the Breitung (2002)⁶) test rejects the null of stationarity and unit root respectively in the Moroccan index. The tests for stationarity and the results for unit roots lead to the conclusion that all consumer price and stock price indices in Egypt, Kenya, Tunisia, Nigeria, and South Africa have at least one unit root. We shall therefore exclude Morocco from the analyses that follow.

4.2. Cointegration

Having established the order of integration, we proceed to apply the Johansen cointegration test. The test is sensitive to the lag length chosen. We therefore estimate a VECM with 12 lags in each case and use Akaike (AIC) and Schwartz (SBC) information criteria to select the appropriate lag. The results from both AIC and SBC are shown in Table 3.

Table 3: Lag length Selection

| lag | Egypt | | Kenya | | Nigeria | | South Africa | | Tunisia | |
|-----|---------|---------|---------|--------|---------|--------|--------------|---------|---------|---------|
| | SC | AIC | SC | AIC | SC | AIC | SC | AIC | SC | AIC |
| 12 | -11.019 | -12.21 | -6.272 | -7.086 | -6.112 | -7.304 | -10.055 | -10.630 | -10.590 | -11.711 |
| 11 | -11.113 | -12.205 | -6.374 | -7.120 | -6.215 | -7.308 | -10.109 | -10.636 | -10.729 | -11.756 |
| 10 | -11.155 | -12.149 | -6.436 | -7.115 | -6.361 | -7.354 | -10.174 | -10.653 | -10.865 | -11.799 |
| 9 | -11.251 | -12.145 | -6.489 | -7.100 | -6.509 | -7.404 | -10.231 | -10.662 | -11.005 | -11.845 |
| 8 | -11.319 | -12.114 | -6.563 | -7.106 | -6.630 | -7.425 | -10.290 | -10.673 | -11.137 | -11.884 |
| 7 | -11.436 | -12.131 | -6.662 | -7.137 | -6.760 | -7.455 | -10.357 | -10.692 | -11.261 | -11.915 |
| 6 | -11.480 | -12.076 | -6.748 | -7.155 | -6.914 | -7.510 | -10.390 | -10.677 | -11.407 | -11.967 |
| 5 | -11.414 | -11.911 | -6.804 | -7.143 | -7.044 | -7.541 | -10.455 | -10.694 | -11.496 | -11.963 |
| 4 | -11.455 | -11.852 | -6.842 | -7.114 | -7.189 | -7.587 | -10.505 | -10.69 | -11.649 | -12.023 |
| 3 | -11.572 | -11.870 | -6.941 | -7.145 | -7.340 | -7.638 | -10.540 | -10.683 | -11.800 | -12.080 |
| 2 | -11.71* | -11.907 | -7.017* | -7.153 | -7.45* | -7.649 | -10.55* | -10.654 | -11.943 | -12.130 |
| 1 | -11.672 | -11.772 | -7.001 | -7.069 | -7.273 | -7.372 | -10.522 | -10.570 | -11.97* | -12.073 |

Note: * indicates lag order selected by the Schwartz criterion

The specification of the cointegration test is as follows: a constant term is restricted in the cointegration space, which allows for a nonzero mean. The lag lengths from the VECM are

⁶ See the technical appendix for a discussion on the Breitung (2002) nonparametric unit root test.

those specified using SBC in Table 3. The results from the trace test based on the Johansen maximum likelihood estimation are provided in Table 4.

Table 4: Johansen trace test

| | Egypt | Kenya | Nigeria | South Africa | Tunisia |
|-----------|---------------------|---------------------|---------------------|----------------------|----------------------|
| H0:rank<= | | | | | |
| 0 | 39.753** [0.000] | 30.878** [0.001] | 21.464 * [0.032] | 35.329 ** [0.000] | 34.744 ** [0.002] |
| 1 | 6.5892 [0.155] | 3.0627 [0.578] | 3.7463 [0.463] | 9.6614 * [0.039] | 11.075 [0.086] |

Note: Trace test probability in []. *p*-values are from Doornik (1998); **, * denotes significance at the 1% and 5% respectively. Lag lengths are based on Table 3.

Table 4 results for Johansen’s trace test determine whether a long-term relation exists between each pair of stock prices and consumer prices. We start with the null hypothesis that there is no cointegrating relation, and if this hypothesis is rejected, we test the hypothesis that there is at most one cointegrating vector. Because there are two variables in each model, we test whether the number of cointegrating vectors is zero, one, or two. As Table 4 shows, the results suggest the existence of one cointegrating vector (or long-run relation) between each pair of indices in four countries. The evidence indicates two cointegrating vectors in South Africa.

We also employ the Breitung (2002) and Breitung and Taylor (2003) nonparametric cointegration test to examine possible deviations from linearity (see Technical Appendix). The latter has a number of advantages: first, the short-run component does not affect the asymptotic null distribution of the test statistic. Secondly, the outcome does not depend on the lag length and the inclusion of a trend or a constant (it is well known that the Johansen procedure is sensitive with regard to these two).

Table 5: Breitung Nonparametric test

| | Egypt | Kenya | Nigeria | South Africa | Tunisia |
|-------------|---------|--------|---------|--------------|---------|
| $r=0 \ r>0$ | 238.62 | 176.91 | 314.77 | 686.03* | 312 |
| $r=1 \ r>1$ | 10.61 | 55.53 | 52.04 | 40.42 | 106.14 |
| | C.V 10% | C.V 5% | | | |
| $r=0 \ r>0$ | 596.2 | 713.3 | | | |
| $r=1 \ r>1$ | 222.4 | 281.1 | | | |

**, * denotes significance at the 1% and 5% respectively.

Table 5 presents the nonparametric tests and show that only in the case of South Africa cointegration is not rejected. Given that the Breitung test does not give us a long-run solution or the possibility of imposing restrictions on the cointegrating vector, we shall concentrate on the Johansen results.

The conclusion from Table 4 that stock prices and consumer prices are cointegrated can be used to test if stock prices have a one-to-one relationship in the long-run with consumer prices. For the 5 countries where both stock prices and consumer prices are statistically significant in the cointegrating vector, we also provide likelihood-ratio tests of the restriction that stock prices and consumer prices are independent (see Table 6).

Table 6: Long Run Relationship between Goods Prices and Stock Prices

| | Cointegrating vectors | Loading | $\theta = 1$ | C.I. |
|--------------|---|--|------------------------------|--------------|
| Egypt | $S_{EG_t} = -3.588 + 0.215^{***} P_{EG_t}$ (3.413) | $\alpha_{EG} = -0.0028$ (0.082) | 27.268 [0.0000] *** | 0.203-0.227 |
| Kenya | $S_{KE_t} = 7.742 + 0.292^* P_{KE_t}$ (1.659) | $\alpha_{KE} = -0.007^{***}$ (5.01) | 16.743 [0.0002] *** | -0.052-0.637 |
| Nigeria | $S_{NI_t} = -0.752 + 0.44^{***} P_{NI_t}$ (6.85) | $\alpha_{NI} = -0.0214^{***}$ (3.057) | 16.202 [0.0003] *** | 0.315-0.565 |
| South Africa | $S_{SA_t} = 8.129 + 2.264^{***} P_{SA_t}$ (6.80) | $\alpha_{SA} = -0.0013^{***}$ (4.33) | 4.2046 [0.1222] ^a | 1.612-2.916 |
| Tunisia | $S_{TU_t} = -4.425 + 0.015^* P_{TU_t}$ (1.666) | $\alpha_{TU} = -0.155^{***}$ (3.780) | 9.6949 [0.0213]** | -0.003-0.033 |

t-statistics errors in () and p-values in []. Note: EG, KE, NI, SA and TU = Egypt, Kenya, Nigeria, South Africa, and Tunisia respectively.

$\theta = 1$ is the restriction that the Fisher coefficient is equal to 1. The formal test of this hypothesis is based on the likelihood ratio (LR) statistic. *, ** and *** indicates significance of the at the 10%, 5% and 1% levels respectively. ^a implies that we fail to reject the tax augmented version of the Fisher hypothesis that the coefficient $\theta \geq 1$. C.I. is the confidence intervals for the long-run point estimates of the cointegrating vector.

Based on equation (9), Table 6 reports the estimates of long-run relations between stock prices and the consumer prices. As shown in Table 6, the estimated point coefficients range from 0.015 to 2.264. In all countries, the sign of the estimated coefficient is positive and statistically significant with the exception of Kenya and Tunisia. This indicates a positive relationship between stock prices and consumer prices. In the case of South Africa, however the lower bound of the latter is greater than unity (1.612). Since we expressed the variables in logarithms, the estimated coefficient in each equation shows the elasticity of changes in stock prices with respect to corresponding changes in consumer prices. For instance, the highest estimated coefficient from Table 6 is 2.26 for South Africa. This means that for every 1% increase in P_t , the JSE Share Index is expected to rise by 2.26% over the sample period.

These results are consistent with previous evidence of positive long-run relationship between consumer and stock prices in other markets (see Anari and Kolari, 2001, Al-Khazali and Pyun, 2004, Luintel and Paudyal 2006). We posit a number of reasons for the results. South Africa has traditionally maintained low inflation rates⁷ over the period of this study. The evolution of the market, especially following the abolition of apartheid in the mid-1990s was accompanied by a significant rise in stock prices following the re-admission of South Africa into the international community. There have also been capital flows following the lifting of sanctions, which may have resulted in boost in equity prices. Thus, low inflation coupled with rising equity prices could explain the large positive relationship between stock prices and consumer prices. Secondly, as argued in many studies, *inter alia*, Darby (1975), Carrington and Crouch (1987) and more recently Crowder and Hoffman (1996); asset holders are liable for paying taxes on their income (e.g. income as well as capital gains). Therefore, for an investor to be

⁷ Single digits compared to Kenya and Nigeria. Following the introduction of inflation targeting by the reserve bank in 2000, rarely has the inflation rate exceeded 6%.

fully compensated for inflation, the nominal return rate should include the effects of both taxes and inflation. Although we do not have reliable estimate of taxes for South Africa, we can argue that the finding of unity elasticity is consistent with the tax-augmented version of the Fisher hypothesis⁸; that is, the return on stocks must exceed the inflation rate to compensate for the loss in real wealth of tax paying investors.

The lower Fisher coefficients for Nigeria, Egypt, Tunisia, and Kenya (insignificant in the last two countries) provide more conservative estimates of how developments in consumer prices affects stock prices in the long-run. Admittedly, the process of emerging has been matched with increasing equity prices. Nearly all the stock markets in our sample have experienced large appreciation in their respective indices over the past decade. Thus, the finding of less than unity for these countries is at variance with the first explanation for South Africa, but not the second. Thus, we do not find evidence of the tax-augmented version of the Fisher hypothesis in these countries. It can also be argued that not only do these markets fail to include information contained in inflation, but also they offer only a partial hedge to investors against rising inflation.

Table 6 also shows the estimates of the speed of adjustment parameters, which indicates how quickly disequilibrium between consumer prices and stock prices is eradicated. These estimates range from 0.0013 to 0.5. Thus, stock prices take a longer time to return to their long-run equilibrium values following movements in goods market prices in South Africa than in Tunisia.

⁸ Our finding for South Africa is in line with studies such as Anari and Kolari (2001), Al-Khazali and Pyun (2004) and Luintel and Paudyal (2006) who estimate Fisher coefficients of above unity.

4.2.1. Stability Analyses

Stability tests are conducted over the sample period for each country. Most of the periods analysed include oil price shocks, emerging market crises and various institutional reforms. Such episodes may induce structural shifts in the long-run relationship between stock prices and consumer prices. This enables us to investigate how the cointegration relationship has changed over time and to identify breaks (see Hansen and Johansen, 1999). Using an expanding window, we calculate the trace test adding one observation at a time. We then divide the trace test with the 5% critical value (obtained from MacKinnon et al, 1999). If this is above one, the null of non-cointegration is rejected. The evidence from Figure 3 indicates that cointegration between commodity and stock prices have been stable throughout the sample for South Africa and Kenya. In the case of Nigeria no cointegration cannot be rejected between 1995 and 1999 and then after 2006. In Egypt, a blip occurred in late 2003 to mid 2005. These indicate the periods at which there has been a significant drift in the relationship between goods prices and stock prices.

Figure 3: Stability of Cointegrating Relationship

The blue line gives the threshold at which cointegration in each market can be accepted or rejected. For instance, if the red line is above the blue line, we cannot reject stable long run relationship between goods prices and stock prices, and the vice versa

Figure 3a: Egypt

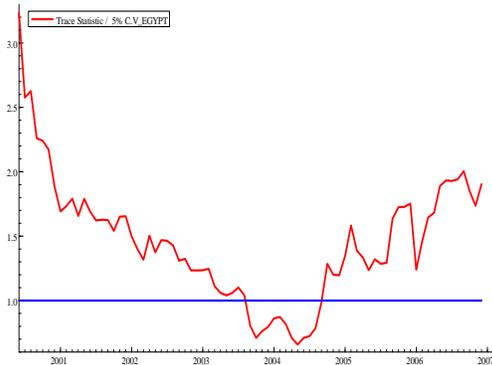


Figure 3b: Kenya

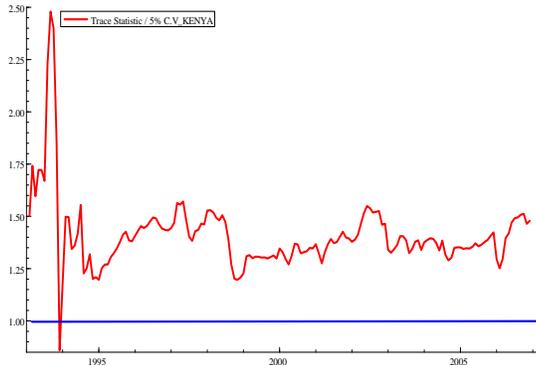


Figure 3c: Nigeria

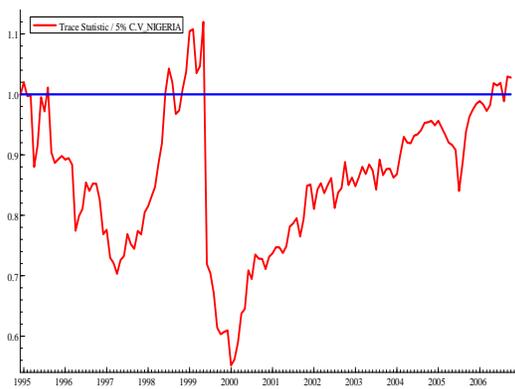


Figure 3f: Tunisia

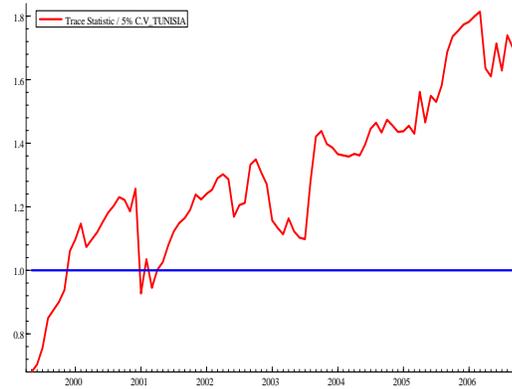


Figure 3e: South Africa



4.3. Time Path of Stock Prices to innovations in goods Prices

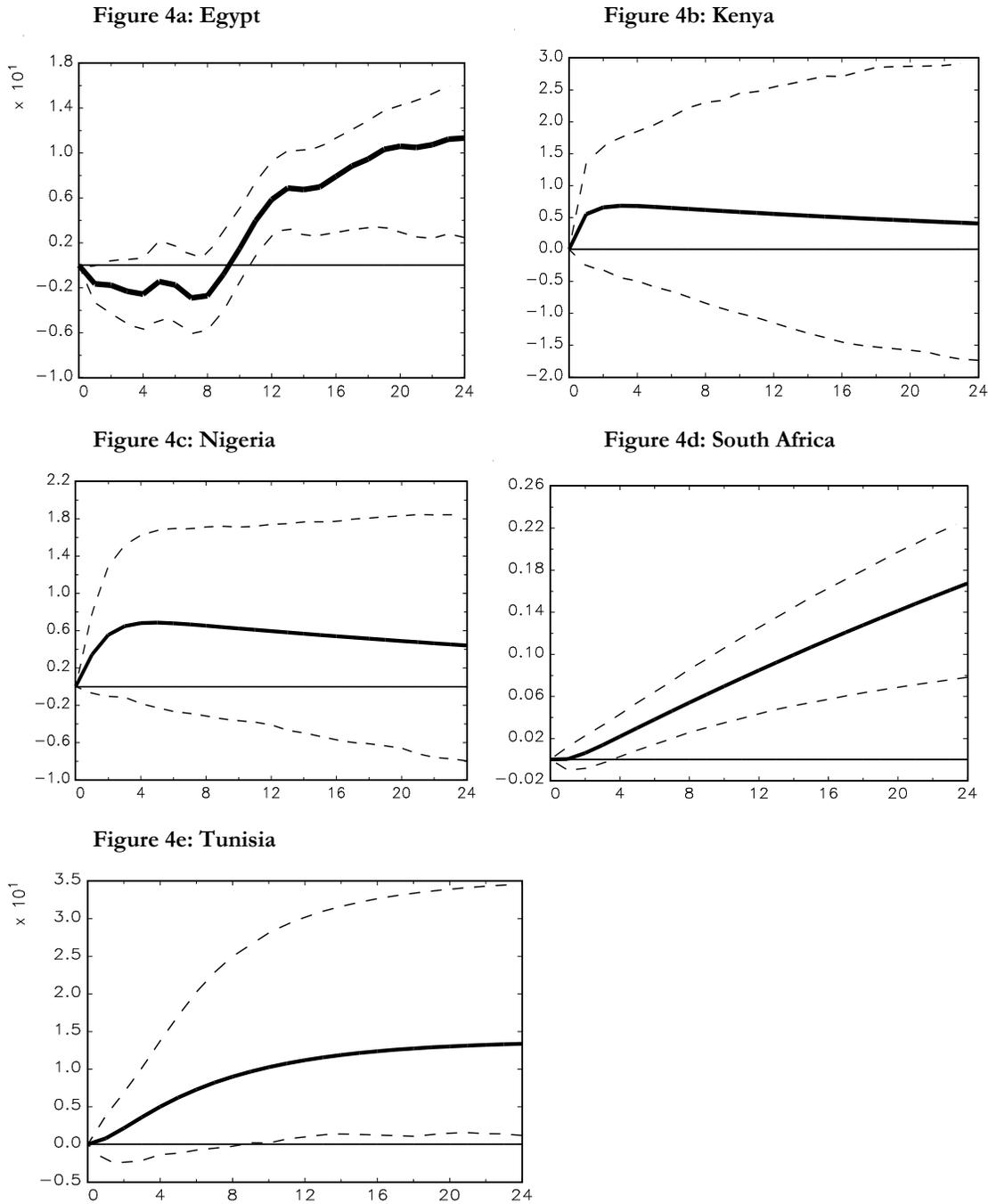
Next, we explore how stock prices react to shocks in consumer prices through impulse response functions. Under the VECM, a shock to a variable directly affects the variable, and is

transmitted to other endogenous variables through the dynamic structure of the VECM (see Johansen, 1995). More specifically, an innovation in the error term in (7) will immediately change the value of current S_t . It will also affect all future values of S_t and changes in P_t . The impulse response functions shed light on the dynamics of the variables included in the VECM system⁹. The response of the stock price indices to unexpected movements in the goods prices with the 95% confidence bands is shown in Figure 4. The figure shows that an unexpected movement in the consumer price index influences the stock price index over time with varying response in each country.

⁹ The impulse response function, or moving average representation, is preferred in this work as opposed to the VAR system because autoregressive systems are very difficult to define succinctly; for instance, there may be complex patterns of cross-equation feedbacks and estimated lagged coefficients that tend to oscillate.

Figure 4: Response of Stock Prices to innovations in Goods Prices

Note: The forecast horizon is 24 months (measured on the horizontal axis). The impulse response function is computed by artificially imposing a one standard deviation shock to one variable and by measuring the response of each variable in the system. The dotted lines indicate the 95% confidence intervals, constructed with 1000 bootstrap replications.



As we can see from the graphs in Figure 4, the initial short-run response of stock prices is negative but insignificant for Egypt (up to 3 quarters) and for South Africa (about a quarter).

The negative short-run relationship between stock prices and consumer prices (also called the inverted Fisher effect) is very prevalent in the literature. However, the evidence from Figures 4a and 4d indicate that such negative responses are only transitory for Egypt and South Africa. The relationship becomes positive as the time horizon increases (after the first three quarters for Egypt, while it is even shorter for South Africa). For Nigeria, Kenya, and Tunisia, the response of stock prices to innovations in consumer price index is invariant to the time horizon. At the 24-month horizon, there exists a positive relationship between stock prices and consumer prices in the three countries. The results from the impulse responses correspond with the previous finding of a positive long-run relationship between consumer prices and stock prices in all the markets. This implies that, at least in the long-run, investors in African markets should expect the stock market to provide a shelter for them against rising consumer prices.

5. Summary and Conclusions

This study examined the relationship between stock returns and inflation in six African countries. We raise the question whether stock investments provide a hedge against inflation in the major African stock markets, especially in the light of the increased attention that they have received from both academics and practitioners. When estimating the long-run generalized Fisher effect stock returns and inflation are calculated using first differences of stock prices and consumer prices leading to significant loss of important information contained in the two series (see Gallagher, 1986). Previous evidence also employ long span of dataset (see Boudoukh and Richardson 1993). However, a long span of data is a perennial problem that researchers face in African stock markets. An alternative approach employed in this study that obviates the need for a long span of data, and preserves the information contained in our series of interest, is to use levels of stock prices and consumer prices. In this

regard, we examine monthly stock price and consumer price indices for a period of min 10 years and a max of 27 using cointegration. The results of the cointegration test support the long-run relationship between stock prices and consumer prices. The long-run generalized Fisher elasticities of stock prices with respect to consumer prices are positive and statistically significant with the exception of Kenya and Tunisia. The point estimates vary from 0.015 for Tunisia to 2.264 for South Africa. We also find that the time path of the response of stock prices to a shock in consumer prices exhibits an initial negative response in Egypt and South Africa, which turns positive over the long-run.

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Appendix
Stock Returns and Inflation in African Countries

| | Egypt | Kenya | Morocco | Nigeria | South Africa | Tunisia |
|----------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Sample | 1997M02 to 2006M12 | 1990M02 to 2006M12 | 1995M02 to 2006M12 | 1991M08 to 2006M10 | 1980M02 to 2007M01 | 1996M01 to 2006M10 |
| α | 1.658(3.48) | 0.486(0.582) | 0.914(2.056) | 2.001(3.446) | 1.032(2.144) | -0.432(-0.782) |
| β | -0.5404(-0.719) | 0.852**(2.23) | 0.394(0.684) | 0.447**(2.410) | 0.084(0.18) | 2.009(1.37) |
| R^2 | 0.004 | 0.024 | 0.003 | 0.032 | 0.000 | 0.014 |
| DW | 1.492 | 2.149 | 1.883 | 1.487 | 1.387 | 1.659 |
| B.G(2) | 4.352[0.015] | 0.576[0.562] | 1.932[0.38] | 15.04[0.00] | 36.23[0.00] | 4.051[0.131] |
| RESET | 4.277[0.233] | 3.160[0.367] | 0.285[0.86] | 2.5779[0.275] | 1.827[0.401] | 0.647[0.723] |
| Wald | | 0.1503[0.698] | | 8.845[0.002] | | |

Note: **, indicates significance at the 5%. DW and B.G is the Dubin-Watson and Breush-Godfrey test for first order and higher order residual correlation. Wald is the Wald coefficient test on the restriction that $(\beta = 1)$ while the RESET test is reported to check any functional misrepresentation of the model. T-statistics in () while p-values reported in [].

TECHNICAL APPENDIX

BREITUNG NONPARAMETRIC TEST FOR COINTEGRATION

Breitung's unit roots and cointegration test employ a variance ratio as the test statistic. As noted, this approach can eliminate the problem of the specification of the short run dynamics and the estimation of nuisance parameters. If $\{y_t\}_1^T$ denotes an observable process that can be decomposed as $y_t = \delta' d_t + x_t$, where $\delta' d_t$ is the deterministic part ($d_t=1$ or $[1, t]'$), and x_t is the stochastic part. If we do not assume the deterministic part, then y_t is consistent with x_t . The null hypothesis is that x_t is $I(1)$, if $T \rightarrow \infty, T^{-1/2} x_{[aT]} \Rightarrow \sigma W(a)$, where $\sigma > 0$ represents the constant (long-run variance), and $W(a)$ denotes a Brownian motion, $[]$ is the integer part. The expression of x_t makes possible the application of a general data generating process. Asymptotically, to construct a consistent estimate, which does not require the specification in short run dynamics, and an estimate of σ , Breitung has proposed the following test statistic

$$\rho = \frac{T^{-4} \sum_{t=1}^T U_t^2}{T^{-2} \sum_{t=1}^T u_t^2} \quad (3)$$

where u_t is the OLS residuals that $u_t = y_t - \delta' d_t$, and U_t is the partial sum process that $U_t = u_1 + \dots + u_t$. If y_t is $I(0)$, the test statistic ρ_T converges to 0. Breitung shows that the variance ratio test has favourable small sample properties using Monte Carlo simulations.

We could proceed and test for cointegration by the generalization of the nonparametric unit roots test on the assumption that the process can be decomposed into a q -dimensional vector of stochastic trend components ξ_t and a $(n-q)$ -dimensional vector of transitory components of v_t where n is the number of variables. Asymptotically, ξ_t and v_t is $T^{-1/2} \xi_{[aT]} \Rightarrow W_q(a)$ and $T^{-2} \sum_{t=1}^T v_t v_t' = o_p(1)$, respectively, where $W_q(a)$ denotes a q -dimensional Brownian motion with unit covariance matrix. The dimension of ξ_t is related to the cointegration rank. In addition, it assumes that the variance of ξ_t diverges with a faster rate than v_t instead of assuming the stationarity of v_t . From the assumption, any process can generate the transitory component denoting the cointegration relationship.

To test the number of cointegrating vectors, Breitung has proposed the following problem about the $n \times n$ matrix A_T, B_T .

$$|\lambda_j B_T - A_T| = 0$$

where $A_T = \sum_{t=1}^T u_t u_t'$, $B_T = \sum_{t=1}^T U_t U_t'$, and $U_t = \sum_{j=1}^t u_j$ represent the n -dimensional partial sum concerning u_t . The problem is equivalent to solving the eigenvalue of $R_T = A_T B_T^{-1}$.

The solution of equation (3) is $\lambda_j = (\eta_j' A_T \eta_j) / (\eta_j' B_T \eta_j)$ where η_j is the eigenvalue of λ_j . If the vectors of the stochastic trends are less than q , $T^2 \lambda_j$ diverges to infinity. In that case, since

stochastic trends are linked with each other, a cointegrating vector exists. Hence, the test statistic is the following.

$$\Lambda_q = T^2 \sum_{j=1}^q \lambda_j,$$

where $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ is the ordered eigenvalues of R_T . The idea of cointegration rank behind the approach is similar to Johansen's idea. The statistic tests whether a q -dimensional stochastic component is rejected at the significance level.